Effects of Worked Examples Using Manipulatives on Fifth Graders’ Learning Performance and Attitude toward Mathematics

Chun-Yi Lee1* and Ming-Jang Chen2
1 Center for Teacher Education, National Taipei University, Taiwan, No.151, University Rd., San-Shia Dist., New Taipei City 23741, Taiwan // 2 Center of General Education, National Chiao-Tung University, Taiwan, No. 1001, University Rd. Hsinchu 30010, Taiwan // chunyi.lii@gmail.com // mjchen@mail.nctu.edu.tw
*Corresponding author

(Submitted August 26, 2013; Revised January 28, 2014; Accepted February 2, 2014)

ABSTRACT
The purpose of this study was to investigate the influence of worked examples using virtual manipulatives on the learning performance and attitudes of fifth grade students toward mathematics. The results showed that: (1) the utilization of non-routine examples could promote learning performance of equivalent fractions. (2) Learning with virtual manipulatives was as effective as with physical manipulatives. (3) Virtual manipulatives can increase learning enjoyment compared with physical counterparts. Implications and recommendations are also presented for future research.

Keywords
Non-routine example, Equivalent fraction, Worked example, Virtual manipulative

Introduction
The concept of equivalence plays an important role in learning fractions. Students must first be equipped with basic concepts related to fractions before they can learn the meaning of equivalent fractions and it is only after these two steps that the concept of rational numbers can be further developed. The equivalence relation of fractions must be taken into consideration when comparing the order relation between two fractions. An understanding of the concept of equivalent fractions is the basis for performing the four arithmetic operations (addition, subtraction, multiplication and division) on fractions with different denominators. Equivalent fractions are the most difficult of all sub-concepts related to fractions, requiring that students have flexibility in their thought processes and a willingness to solve problems by advancing from concrete operations into formal operations. However, among elementary school students, the comprehension of the concept of equivalent fractions tends to be quite weak (Kamii & Clark, 1995). Elementary school students in Taiwan also have this problem; in fact, many students are unable to fathom that one half is equal to two quarters, even if they have previously studied equivalent fractions (Yu & Leu, 2002). An incomplete understanding of fractions or overly rigid thinking can prevent students from solving problems related to equivalent fractions.

Most elementary school students are at the stage of concrete operations, in which action and iconic representation can only be formed through actual practice. Therefore, physical manipulatives are commonly used in mathematics education to make abstract ideas and symbols more meaningful and comprehensible to students (Durmus & Karakirik, 2006). Developing a comprehension of concepts related to equivalent fractions requires that objects be split into different portions; however, physical manipulatives are ill-suited to the arbitrary splitting of objects. Moyer, Bolyard, and Spikell (2002) recently defined virtual manipulatives as interactive, web-based visual representations of a dynamic object capable of facilitating the development of mathematical concepts by students. Virtual manipulatives allow the splitting of objects into different portions to facilitate the learning of concepts associated with equivalent fractions. This enables students to visualize specific, abstract mathematical concepts and the content of learning can be broken down for presentation purposes (Chang, Yuan, Lee, Chen, & Huang, 2013). This makes virtual manipulatives excellent tools to facilitate the learning of concepts associated with equivalent fractions.

Worked examples have been successfully applied in the instruction of computer programming, algebra, and geometry (Carroll, 1994; Paas & van Merrienboer, 1994). Students with experience using worked examples as an instructional strategy adopt problem-solving techniques more quickly and present heightened problem solving performance (Chandler & Sweller, 1991). The provision of appropriate worked examples can aid in the formation of concepts by learners. Virtual manipulatives and worked examples offer exciting new possibilities as learning aids; however, there’s a noticeable lack in the literature of the large-scale validation of these strategies and few studies have...
explored the practical implementation of combining worked examples with virtual manipulatives. Understanding the properties of worked examples with virtual manipulatives and how these relate to learning is important in predicting which material will be more beneficial and can help to inform the design of new learning materials. This study explores how different approaches to example integration influence the learning performance of fifth grade students.

**Literature review**

**Factors affecting the learning of equivalent fractions**

A failure to learn the concept of equivalent fractions is due mainly to overly rigid thinking (Peng & Leu, 1998). Many factors influence the learning of equivalent fractions, such as the capacity for flexible thinking, combining, operational thinking, and unit forming, as outlined in the following.

In the graphic representation of continuous quantity, flexible thinking capability refers to the ability of learners to realize that fractions with different names are actually the same fraction (for example: 2/6 and 3/9 both equal 1/3) through visualization and ignorance of the split line. The ability to perform these procedures is largely determined by the number of split lines and whether the split regions are interconnected. Many students insist that they can only accept an equivalent name for a fraction when the denominator of the fraction is equal to the number of split blocks and when the split blocks are interconnected (Behr, Wachsmuth, Post, & Lesh, 1984; Behr & Post, 1992). Booth (1987) interviewed 11-year-old school students in Britain and found that 95% of them believed that Figure A in Figure 1 was 1/3, while 73% of them thought that Figure B was 1/3; a difference of 22%. The problem was that the students believed that 2/6 is different than 1/3. This is an indication that if the learner has a graphic image of the problem or learns to ignore the split line, another name can be generated for the equivalent fraction.

In the graphical presentation of discrete quantities, flexible thinking capability refers to a learner’s ability to utilize actual manipulatives or their imagination to conduct re-partition or re-combination in the process of problem solving (Behr et al., 1984). For example, when we use small circles to solve the problem of 2/3=4/6, the first thing to do is convert the figures into fractions. First, each pair of circles in Figure 2 is regarded as one set, such that six circles become three sets. Four of the six circles are black, so they can be converted into the fraction 4/6. Two of the three sets of circles are black, so they can be converted into the fraction 2/3. There are exactly the same numbers of black circles, leading to the result that 4/6=2/3.

“Combination capability” refers to the students’ ability to use specific problem solving strategies to deal with problems of equivalent fractions or residual fractions. Students demonstrate this capability by dividing a unit quantity into several parts and applying the correct treatment to every part. Combination capability is the ability to combine every treated quantity into a designated fraction of unit quantity (Peng & Leu, 1998). “Operational thinking capability” refers to the ability to use different splitting approaches on the same figure, i.e., a situation in which there are equal areas, but different shapes (Kamii & Clark, 1995).

Knowledge can be classified into the figurative and operative aspects. The figurative aspect of knowledge is based on the observed shape of knowledge, while the operative aspect is based on relation knowledge, a process which cannot be observed. For example, there are different ways to split the rectangle into two halves, resulting in either rectangles
In this manner, the student either or whether or not can find an appropriate unit within the figures to precisely split the designated part in order to solve problems of equivalent fractions. The reverse capability is the ability to use the unit to recombine the figure into a whole entity after the exact splitting of the appropriate units; this is called the unit forming capability (Saenz-Ludlow, 1994, 1995). For example, Saenz-Ludlow (1994, 1995) provided several b1, b2 and b3 figures (shown in Figure 3) to third-grade students. At first, the students were allowed to compare b1 with the whole entity and then compare b2 and b3, respectively. Using these three pieces, students were able to divide the whole entity (Figure 3), such that b2 is 1/3 of the whole entity. In this study, when children were first asked to compare b1 with b2, they could not find the correlation between them; however, after receiving a hint using b3, they quickly realized that b2 is 2/3 of b1. This indicates that the students knew that every small unit must be equal to each other after splitting, which enabled them to adjust the small units in order to come to a solution. When b3 was presented, the students said that b2 is 2/3 of the half piece (b1). B3 gave them a representation of the relationship between b1 and the whole entity, and 2/3 represents the relationship between b2 and b1.

Figure 3. Illustration of unit forming with equivalent fractions

Current status of teaching materials for equivalent fractions

As indicated by the mathematics curriculum standards of the Nine Year Integrated Curriculum Syllabus in Taiwan, basic fractions are introduced from first grade to third grade, while the meanings of equivalent fractions, fraction reduction/expansion, and the addition/subtraction/multiplication/division of fractions are introduced from fourth to fifth grades (Taiwan Ministry of Education, 2004).

In Taiwan, all versions of equivalent fraction teaching materials are presented without the direct use of terms such as fraction expansion and fraction reduction, and without the direct introduction of an operation approach wherein the value of a fraction remains unchanged, with both sides of the equations multiplied or divided by the same number. Instead, smaller unit components are generated through re-splitting or re-combination in specific scenarios, or larger unit components are generated through merging or re-combination, while the equivalent relationships of components represented by these two kinds of unit components are compared to each other. Under the scenario of continuous quantity, two strategies can be adopted: resorting to intuitive experience and resorting to the number of split parts. Resorting to intuitive experience refers to the approach of splitting ribbons or pies into different parts and comparing the lengths and sizes of different quantities to recognize the equivalence of two fractions in terms of length or area and concluding that the two fractions are equal. Resorting to the number of split parts refers to the direct notification of the number of split parts and determination of whether or not the two fractions are equal to each other with the approach of number line. Under scenarios of discrete quantity, the strategies of “resorting to the content” and “resorting to merger or re-combination of the content” are most commonly adopted for problem solving. Resorting to the content refers to the determination of equivalence through a comparison of quantity of contents. Resorting to merger or re-combination of the content refers to the approach of finding other names for the fraction by regarding multiple contents as a single entity and determining whether the quantities are identical.

Analyzing the content of the Taiwanese teaching materials for equivalent fractions led us to the conclusion that the existing examples addressed the issue of flexible thinking in continuous quantity and discrete quantity scenarios. The examples allowing flexible thinking in scenarios of continuous quantity include interconnected split blocks, while the examples based on scenarios of discrete quantity deal with activities involving the splitting or combining of groups of identical objects. Thus, providing worked examples of non-continuous colored blocks in the scenario of
continuous quantity, and demonstrating the existence of more than two kinds of objects in equivalent fraction problems in group mode, may be effective techniques with which to improve the flexible thinking ability of students.

Virtual Manipulatives

A virtual manipulative is very similar to a physical manipulative. It is a dynamic object with interactive features, which can be put on a website. The presentation of this kind of dynamic object provides students with an opportunity to construct mathematical knowledge (Moyer, Bolyard, & Spikell, 2002; Moyer, Niezgoda, & Stanley, 2005). Virtual manipulatives are equipped with the following features (Yuan, Lee, & Wang, 2010; Chang, Yuan, Lee, Chen, & Huang, 2013) : (1) variability, in which the learner can color parts of the objects, or increase or reduce the quantity of certain object; (2) unlimited supply, which resolves the issue of an insufficient number of physical manipulatives in class; virtual manipulatives also save teachers from the time consuming distribution and organization of teaching aids, and makes it easy to arrange teaching aids simply by clicking the recycle bin icon to clear all teaching aids from the screen; (3) simultaneously presentation of figures and symbols on the screen to enhance the link in the learner’s mind between these two presentations of quantity.

Empirical evidence related to the use of virtual manipulatives for mathematics learning in the classroom is still relatively new and somewhat limited. Yuan, Lee, and Wang (2010) examined the effects of using virtual manipulatives and physical manipulatives in the study of polyominoes on junior high school students. In that study, the group using virtual manipulatives learned as effectively as the group using physical manipulatives. In addition the use of virtual manipulatives enabled the generation of new ideas and helped to develop an appreciation of symmetry and the effectiveness of rotating figures. Manches, O’Malley, and Benford (2010) indicated that differences in the properties of manipulatives (comparing virtual and physical manipulatives) might influence the numerical strategies employed by students. Suh and Moyer (2007) examined the effects of developing representational fluency using virtual and physical manipulatives. The results showed that despite differences in manipulative models, both the physical and virtual environments are effective for learning and encouraging relational thinking and algebraic reasoning.

In contrast, Chang et al. (2013) adopted a non-equivalent quasi-experimental design, and recruited participants from two classes of third grade students in an elementary school in Taiwan. Their results demonstrated the effectiveness of virtual manipulatives over that of physical manipulatives on three subscales of immediate learning performance and all four subscales of retention performance. Remier and Moyer (2005) reported that third grade students using virtual manipulatives to learn fractions showed statistically significant gains in the development of conceptual knowledge. Student surveys and interviews have indicated that manipulatives capable of providing immediate and specific feedback are easier to use than traditional methods, and enhance enjoyment while learning. These studies demonstrated that virtual manipulatives offer unique advantages and can be as or more effective than physical manipulatives in the support of learning. However, it is more important to examine the characteristics of learning environments using manipulatives and how these characteristics influence learning experiences. Therefore, this study explored the effects of various worked examples using virtual or physical manipulatives on the learning of equivalent fractions.

Method

Research design

This study adopted a quasi-experimental design with the “method of example integration” as an independent variable. Based on differences between worked examples using virtual or physical manipulatives, example integration can be categorized as traditional continuous examples (TCE), technology supported continuous examples (TSCE), and technology supported mixed examples (TSME). The dependent variables were “the learning performance of equivalent fractions” and “mathematics attitudes.” The learning performance of equivalent fractions refers to the learning outcomes after the experiment: (1) basic flexible thinking: basic inpainting capability and splitting capability; (2) advanced flexible thinking: advanced inpainting capability, combination capability, operational thinking capability, and unit forming capability. Attitude toward mathematics learning refers to the
learners’ perception of mathematics following the experimental teaching, including learning enjoyment, learning motivation, and reported anxiety caused by studying mathematics.

Participants

The participants in this study included 5th grade students from an elementary school in Taipei City. All of the participants had learned basic concepts related to fractions, such as decimals, simple fractions, and unit quantity before this teaching experiment was conducted. As such, they were equipped with preliminary knowledge of equivalent fractions but without having learned the concept of equivalent fractions. In order to coordinate with the curriculum of the original class, we randomly selected 100 participants from three classes out of 13 classes in the 5th grade, and randomly assign 34 students for the TCE group, 32 students for the TSCE group, and 34 students for TSME group. The sample was 90 students after eliminating learners who failed participate fully. This left 30 students left in the TCE group, 30 students left in the TSCE group, and 30 students in the TSME group.

Instruments

Teaching materials

The main purpose of this study was to explore the effects of various approaches to example integration on the learning of equivalent fractions by students in the 5th grade. These approaches to example integration can be categorized as “continuous examples” and “mixed examples.” Continuous examples refer to continuous problems of equivalent fractions, while mixed examples refer to solving discontinuous and continuous problems of equivalent fractions. The various approaches to teaching were “traditional instruction” and “technology supported instruction.”

Traditional instruction refers to the instructor explaining the concept of equivalent fraction in conjunction with physical manipulatives and providing students with opportunities to use manipulations to verify mathematical concepts. Technology supported instruction refers to the teacher explaining the concept of equivalent fractions in conjunction with virtual manipulatives and providing students with appropriate opportunities to use manipulations to verify the mathematical concepts. Virtual manipulatives in this study included a “Magic Board” and “Fraction Bar” developed by Yuan & Lee (2012). Magic Board maintains the properties of physical manipulatives and enables a clearer representation of mathematical concepts. This tool is available on the internet (http://163.21.193.5). Another virtual manipulative is the Fraction Bar, which is an interactive tool used to explore the concepts of fractions in length mode.

According to the approach adopted for example integration, the students were divided into TCE, TSCE, and TSME groups. The difference between the TCE and TSCE groups was the use of the different manipulatives, in which TCE group verified the concepts of equivalent fractions using physical manipulatives and the TSCE group verified concepts of equivalent fractions using virtual manipulatives. These two groups had identical examples and approaches to the explanation of concepts. The differences between the teaching materials used in TSCE and TSME are listed in the following:

- **TSCE**

In a continuous quantity scenario, continuous examples refer to colored blocks that are continuous and non-scattered, such that the learners can refer to the same fraction using different names through visualization and ignorance of the split line. Examples of continuous equivalent fractions include equivalent fraction problems in the length mode and area mode. Take for example the length mode, as shown in Figure 4. In order to identify a fraction that is equivalent to 1/2, the learner can use the virtual manipulative (fraction bar) to figure out the appropriate splitting approach and then apply color to find a colored ribbon that is equivalent to 1/2 of the colored ribbon. The fraction bar can only color the figure by dragging, which prevents the scattering of the colored blocks. We can use the alignment line to ensure that every fraction figure has the same length.

In discrete quantity scenarios, continuous examples refer to the equivalent fraction problem in group mode, in which re-splitting or re-combination is conducted using a group of identical objects to demonstrate that two fractions are
equal. A comparison between the quantity of 2/5 a box of apples and 6/15 a box of apples is as shown in Figure 5. Virtual manipulative can be used to reduce the number of split lines in order to identify the most appropriate splitting approach and to provide a coloring function to help students realize these two fractions are equal.

- **TSME**

In addition to providing learners with examples of continuous equivalent fractions, mixed examples can also provide non-routine examples that cannot be seen in traditional teaching materials. In the scenario of continuous quantity, non-routine examples refer to colored blocks that are not continuous, in which the learner must first arrange discontinuous blocks into continuous blocks and refer to the same fraction using different names through visualization and ignorance of the split line. Non-routine examples of equivalent fractions in the scenario of continuous quantity include problems of equivalent fractions in length mode and area mode. Take the non-routine example of the equivalent fraction of length mode, as shown in Figure 6. To obtain the answer for 1/3=/?/6, the learner can use virtual manipulatives to add or eliminate split lines, identify an appropriate splitting approach, color any small frame of the fraction bar with six equal portions, move all discontinuous blocks together to form continuous blocks as a figure of the fraction, and arrange the fraction bar into three equal portions to verify that 1/3 is equal to 2/6.
In discrete quantity scenarios, non-routine examples refer to situations in which there are more than two kinds of objects in the equivalent fraction problem in group mode. The learner must first re-arrange the objects to determine an appropriate splitting approach in order to realize that the two fractions are equal. In the example in Figure 7, finding the equivalent fraction of 16/24 involves scattered objects, such that the learner must first move the figures and re-arrange them into a single object. The coloring function is then used to identify an appropriate splitting approach for drawing split lines and regarding multiple objects as a group to understand that the two fractions are equal. In this manner, the students obtain an equivalent fraction.

Equivalent Fraction Achievement Test

The purpose of the equivalent fraction achievement test was to evaluate learning performance with respect to equivalent fractions after conducting teaching experiments involving different approaches to example integration. The test was meant to evaluate Taiwanese teaching materials related to equivalent fractions. The test items were used to measure two types of thinking, “basic flexible thinking” and “advanced flexible thinking.” Basic flexible thinking involves the basic thought processes involved in resolving equivalent fractions, divided into basic inpainting capability and splitting capability. Basic inpainting capability is a measure of the learner capacity associated with solving continuous equivalent fractions under scenarios of continuous quantity, used to determine whether the learner has the flexible thinking. Splitting capability measures students’ ability to solve continuous equivalent fractions in scenarios of discrete quantities, as a reflection of the thought processes involved in re-splitting or re-combining. Advanced flexible thinking involves solving problems associated with discontinuous equivalent fractions. This advanced form of flexible thinking includes four categories: advanced inpainting, combination, operational thinking, and unit forming. Advanced inpainting involves the solving of discontinuous equivalent fractions under scenarios of continuous quantity, regardless of whether the learner can assimilate discontinuous blocks or employs flexible thinking. Combination involves solving problems associated with discontinuous equivalent fractions in scenarios of discrete quantity, as a reflection of the thought processes required for the re-arrangement of more than two kinds of objects as well as re-splitting or re-combining. Operating thinking involves solving problems of equivalent fractions associated with objects of various shapes (i.e., the problem of equal areas within different shapes), and the inference that all figures are equal, despite visual clues to the contrary. Unit forming involves solving problems associated with the precise division of the whole entity using appropriate units, and using these units for recombination into a whole entity. This primary involves the identification of a unit suitable for the division of the object into designated parts.

The items in the achievement test for problems of equivalent fractions measure two types of thinking: basic flexible thinking and advanced flexible thinking. Basic flexible thinking type can be further divided into two categories and advanced flexible thinking can be further divided into four categories, resulting in a total of six forms of flexible thinking capabilities. The test included four problems for each of these capabilities, resulting in a total of 24 problems. One point was assigned for each problem (24 points in total). The reliability of these problems was qualified using a test of internal consistency. The values for Cronbach’s alpha were as follows: basic inpainting (α = 0.70), splitting (0.75), advanced inpainting (α = 0.65), combination (α = 0.83), operational thinking (α = 0.60), and unit forming (α = 0.84). The score for the entire test was α = 0.92, which is an acceptable coefficient of internal consistency. The difficulty of the problems was between 0.57 ~ 0.83, and the item discrimination index was between 0.33 ~ 0.86. These results demonstrate that the achievement test for problems of equivalent fractions has appropriate difficulty and discrimination index.

Questionnaire of Attitudes toward Mathematics Learning

The purpose of the Questionnaire of Attitudes toward Mathematics Learning was to reveal the feelings of learners related to learning equivalent fractions through various approaches to example integration. The content of the questionnaire covered three categories: learning enjoyment, learning motivation and anxiety caused by studying mathematics. A total of 15 problems (five problems for each dimension) were included to deal with attitudes toward mathematics learning. This included positive items for learning enjoyment and learning motivation, and negative items for the anxiety caused by studying mathematics. A five-point Likert Scale was adopted. Scoring for positive items involved awarding 1 point (strongly disagree) to five points (strongly agree); scoring for negative items was just the opposite. The reliability of this questionnaire was verified using an internal consistency test, the values of which are as follows: Cronbach’s alpha for learning enjoyment (α = 0.83), learning motivation (α = 0.85), anxiety
caused by studying mathematics ($\alpha = 0.72$), and for the entire questionnaire ($\alpha = .90$), indicating excellent internal consistency.

**Results**

**Analysis of learning performance in equivalent fractions**

**Basic flexible thinking**

This study employed multivariate analysis of covariance to explore the performance of basic flexible thinking in the solving of equivalent fractions, the results of which are presented in Table 1. The effects of example integration reached the level of significance ($\text{Wilks' Lambda} = .749$, $p < .001$, $\eta^2 = .134$, Cohen $d=.787$) indicating that as far as the performance of basic inpainting and splitting, a significant difference was observed in the average score of at least one capability in groups using different approaches to integration, as shown in Table 1. The influence of example integration method on basic inpainting capabilities was significant ($F_{(2,83)} = 10.106$, $p < .001$), and the influence of example integration method on splitting capabilities was non-significant ($F_{(2,83)} = 1.975$, $p = .145$), indicating that the basic inpainting capabilities of learners receiving TSME ($M = 3.513$) was significantly better than those receiving TCE ($M = 2.662$) and TSCE ($M = 2.825$), and no significant difference was observed in the splitting capabilities of learners using various approaches to example integration.

**Table 1. Summary of covariance analysis on performance of basic flexible thinking for equivalent fractions**

<table>
<thead>
<tr>
<th>Source of variance</th>
<th>Basic flexible thinking</th>
<th>$SS$ (Type III sum of squares)</th>
<th>$df$ (Degree of freedom)</th>
<th>$MS$ (Sum of squares)</th>
<th>$F$ (F test)</th>
<th>Sig. (Significant)</th>
<th>Post comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior knowledge</td>
<td>Basic inpainting</td>
<td>40.329</td>
<td>1</td>
<td>40.329</td>
<td>66.498$^*$</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>capability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Splitting</td>
<td>46.953</td>
<td>1</td>
<td>46.953</td>
<td>46.776$^*$</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>Example</td>
<td>Basic inpainting</td>
<td>12.258</td>
<td>2</td>
<td>6.129</td>
<td>10.106$^*$</td>
<td>.000</td>
<td>TSME $&gt;$ TCE</td>
</tr>
<tr>
<td>integration</td>
<td>capability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Splitting</td>
<td>3.965</td>
<td>2</td>
<td>1.983</td>
<td>1.975</td>
<td>.145</td>
<td>=TSCE</td>
</tr>
<tr>
<td>approach</td>
<td>Basic inpainting</td>
<td>50.337</td>
<td>83</td>
<td>.606</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>capability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Splitting</td>
<td>83.314</td>
<td>83</td>
<td>1.004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>Basic inpainting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>capability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Splitting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$p < .05$.

**Analysis on the advanced flexible thinking performance of equivalent fraction**

This study employed multivariate analysis of covariance to explore the performance of advanced flexible thinking in solving equivalent fraction. The influence of the example integration method was significant ($\text{Wilks' Lambda} = .778$, $p = .009$, $\eta^2 = .118$, Cohen $d=.732$), indicating that there was a significant difference in the average score for at least one capability in each group of the sample. These differences demonstrate that learners using various approaches to example integration differed in their advanced flexible thinking required for inpainting, combination, operational thinking, and unit forming.

The advanced flexible thinking required to solve equivalent fractions includes advanced inpainting, combination, operational thinking, and unit forming. A summary of covariance analysis is presented in Table 2. The effects of advanced inpainting ($F_{(2,83)} = 9.096$, $p < .001$) and operational thinking ($F_{(2,83)} = 4.691$, $p = .012$) reached the level of
significance, indicating that these capabilities were stronger among learners receiving TSCE, compared to those receiving TCE or TSCE. No significant differences were observed between those receiving TCE and TSCE in the performance of advanced inpainting or operational thinking. The effects of the various approaches to integration on combination capability and unit forming capability were non-significant (combination capability: $F_{(2,83)} = 1.311, p = .275$; unit forming capability: $F_{(2,83)} = 2.137, p = .124$).

Table 2. Summary of covariance analysis on performance of advanced flexible thinking for equivalent fraction

<table>
<thead>
<tr>
<th>Source of variance</th>
<th>Advanced flexible thinking capability items</th>
<th>SS (Type III sum of squares)</th>
<th>df/ (Degree of freedom)</th>
<th>MS (Sum of squares)</th>
<th>$F$ (Sum of squares)</th>
<th>Sig. (Significant)</th>
<th>Post comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior knowledge</td>
<td>Advanced inpainting capability</td>
<td>30.703</td>
<td>1</td>
<td>30.703</td>
<td>30.692*</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Combination capability</td>
<td>70.566</td>
<td>1</td>
<td>70.566</td>
<td>89.327*</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Operational thinking capability</td>
<td>.016</td>
<td>1</td>
<td>.016</td>
<td>.017</td>
<td>.897</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit forming capability</td>
<td>59.096</td>
<td>1</td>
<td>59.096</td>
<td>36.431*</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>Example integration approach</td>
<td>Advanced inpainting capability</td>
<td>18.199</td>
<td>2</td>
<td>9.100</td>
<td>9.096*</td>
<td>.000</td>
<td>TSME&gt;TC E=TSCE</td>
</tr>
<tr>
<td></td>
<td>Combination capability</td>
<td>2.072</td>
<td>2</td>
<td>1.036</td>
<td>1.311</td>
<td>.275</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Operational thinking capability</td>
<td>8.875</td>
<td>2</td>
<td>4.438</td>
<td>4.69*</td>
<td>.012</td>
<td>TSME&gt;TC E=TSCE</td>
</tr>
<tr>
<td></td>
<td>Unit forming capability</td>
<td>6.935</td>
<td>2</td>
<td>3.467</td>
<td>2.137</td>
<td>.124</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>Advanced inpainting capability</td>
<td>83.030</td>
<td>83</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Combination capability</td>
<td>65.568</td>
<td>83</td>
<td>.790</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Operational thinking capability</td>
<td>78.517</td>
<td>83</td>
<td>.946</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit forming capability</td>
<td>134.637</td>
<td>83</td>
<td>1.622</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$p < .05$.

Analysis of Attitudes toward mathematics learning

To uncover the attitudes toward learning mathematics (after the experiment), this study conducted multivariate analysis of covariance with respect to the perceptions of learners in three dimensions: learning enjoyment, learning motivation and anxiety caused by studying mathematics. The influence of the example integration method on attitudes toward mathematics was non-significant (Wilks' Lambda = 1.343, $p = .241$, $\eta^2 = .047$, Cohen $d = .444$).

We also conducted analysis of covariance, as shown in Table 3. The influence of the example integration method on learning enjoyment was significant ($F_{(2,83)} = 3.815, p = .026$), indicating that learning enjoyment was greater among learners receiving TSCE ($M = 4.03$) than those receiving TCE ($M = 3.43$) or TSME ($M = 3.59$). No significant differences were observed in the learning enjoyment among learners receiving TCE or TSME. The influence of the example integration method on learning motivation and anxiety caused by studying mathematics were not significant (learning motivation: $F_{(2,83)} = .242, p = .786$; anxiety caused by studying mathematics: $F_{(2,83)} = .182, p = .834$).
Table 3. Summary of covariance analysis on attitudes toward mathematics learning

<table>
<thead>
<tr>
<th>Source of variance</th>
<th>Attitudes toward mathematics learning</th>
<th>SS (Type III sum of squares)</th>
<th>df (Degree of freedom)</th>
<th>MS (Sum of squares)</th>
<th>$F$ (F test)</th>
<th>Sig. (Significant)</th>
<th>Post comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior knowledge</td>
<td>Learning enjoyment</td>
<td>.026</td>
<td>1</td>
<td>.026</td>
<td>.033</td>
<td>.855</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Learning motivation</td>
<td>.001</td>
<td>1</td>
<td>.001</td>
<td>.002</td>
<td>.969</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Anxiety caused by studying mathematics</td>
<td>.572</td>
<td>1</td>
<td>.572</td>
<td>.882</td>
<td>.351</td>
<td></td>
</tr>
<tr>
<td>Example integration approach</td>
<td>Learning enjoyment</td>
<td>5.901</td>
<td>2</td>
<td>2.950</td>
<td>3.815*</td>
<td>.026</td>
<td>TSCE&gt; TCE= TSME</td>
</tr>
<tr>
<td></td>
<td>Learning motivation</td>
<td>1.377</td>
<td>2</td>
<td>.688</td>
<td>1.293</td>
<td>.280</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Anxiety caused by studying mathematics</td>
<td>1.184</td>
<td>2</td>
<td>.592</td>
<td>.913</td>
<td>.405</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>Learning enjoyment</td>
<td>64.187</td>
<td>83</td>
<td>.773</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Learning motivation</td>
<td>44.207</td>
<td>83</td>
<td>.533</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Anxiety caused by studying mathematics</td>
<td>53.855</td>
<td>83</td>
<td>.649</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$p < .05$.

Discussion and conclusions

This study developed materials specific for teaching concepts related to equivalent fractions. Learning performance and attitudes toward mathematics were compared among students who had received TCE, TSCE, and TSME. Students in the TSME group benefited more than those in the TCE or TSCE groups with regard to basic inpainting, advance inpainting, and operational thinking. These results are in line with those presented by Lee and Chen (2009), indicating that the utilization of non-routine examples can promote learning performance in equivalent fractions. This may be explained by the fact that these non-traditional examples are an effective means to engage the intellect of students, capture their interest and curiosity, develop their mathematical understanding and reasoning processes, and allow for a variety of solution strategies, solutions, and representational forms. Unfortunately, elementary school textbooks in Taiwan present only traditional examples of equivalent fractions. To improve learning performance in equivalent fractions, alternative instruction models should be integrated into the curriculum and students should be encouraged to use non-routine examples.

The difference in learning performance between TCE and TSCE groups was not statistically significant. These results are consistent with those of Yuan, Lee, and Wang (2010), indicating that using virtual manipulatives can be as effective as using physical manipulatives. In other words, the form of the manipulatives makes little difference as long as the method of instruction is preserved, such that replacing the physical materials with virtual materials does not affect learning performance. In fact, our study found that the use of non-routine examples is the most important factor influencing the learning of equivalent fractions. Regardless of the instructional aid used, teachers should pay attention to the arrangement of worked examples in the application of virtual or physical manipulatives. Inappropriate examples can prevent students from learning how to solve problems associated with equivalent fractions.

Students in the TSCE group presented a more positive attitude toward learning mathematics, compared to those in the TCE and TSME groups. This echoes Reimer and Moyer’s (2005) study in which most students responded positively to the use of virtual manipulatives. However, students in TSME group did not describe greater learning enjoyment than those in TCE group, perhaps due to the difficulty and complexity of non-routine examples, compared with continuous examples. Thus, the design of worked examples should be considered an important factor affecting the attitudes of students when using virtual manipulatives.

In summary, this study found that learning outcomes with respect to equivalent fractions can be enhanced by the utilization of non-routine examples. We suggest that appropriate worked examples be provided during teaching, and virtual manipulatives be applied carefully in the instruction of mathematics in order to enhance learning enjoyment.
This study has a number of limitations. First, the sample size was small; therefore, these findings are not necessarily generalizable to other groups of learners with different educational or cultural backgrounds. Second, courses on equivalent fractions differ considerably from those of other domains such as biology or social sciences; therefore, our conclusions cannot be generalized to other disciplines. Finally, prior experience in areas such as mathematics epistemology or computer self-efficacy may influence learning outcomes when using virtual manipulatives or physical manipulatives. Future studies would need to examine the role of prior experience on learning with virtual manipulatives.

Acknowledgements

The funding of this study was supported by the National Science Council, Taiwan under grant NSC 101-2511-S-305-001-, NSC 102-2511-S-305-001-, NSC 103-2511-S-305-001-, NSC 100-2511-S-009-006-, and NSC 101-2511-S-009-006-MY2.

References


