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Mobile-Device-Supported Problem-Based Computational Estimation Instruction for Elementary School Students

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ABSTRACT

This study implemented a three-stage problem-based estimation instruction scenario and combined it with mobile technology to provide elementary teachers with an effective e-tool for observing student estimation and leading effective class or group discussions on the selection and assessment of appropriate strategies for solving daily estimation problems. Twenty-eight fourth graders were randomly sampled and assigned to two groups: the experimental group (problem-based estimation instruction using mobile devices) and the control group (problem-based estimation instruction without mobile devices). The analytical results demonstrated that problem-based estimation instruction could effectively help students learn computational estimation skills. Moreover, using mobile devices for problem-based computational estimation instruction appeared to help students discuss and cooperate with others, and moreover the mobile-device-supported problem-based estimation scenario helped students develop metacognition knowledge of estimation strategies.

Keywords

Computational Estimation, Number Sense, Mobile Learning, Problem-Based Learning

Introduction

Computational estimation refers to attempts to make reasonable guesses of approximate answers to arithmetic problems, either without performing actual calculations or before performing them (Dowker, 1992). Computational estimation is a complex skill involving many of the same subtleties and complexities as problem solving. A good estimator can select a strategy appropriate to the problem, including the specific numbers and operations involved (Reys, 1986). Abilities such as flexible thinking, decision-making, answer adjustment, and filtering out of non-sensible answers are crucial to cultivating good estimation skills (Trafton, 1988) and good estimation skills can in turn enhance applied mathematical skills (Coburn & Shulte, 1986).

Computational estimation is one of the most powerful and useful aspects of estimation, and becomes easier to use in daily life, where an estimate is often the only alternative to solving a problem (for example, rapidly selecting the best bargain in a time-limited anniversary sale). The most widely used computational estimation strategies are rounding (up or down), compatible numbers, front-end, and clustering. The rounding strategy is a powerful and efficient method of estimating the sum of several numbers or the product of two multi-digit factors. The rounding strategy can round numbers to the closet larger decade (rounding-up) (e.g. calculating $90 \times 80 = 7200$ to estimate $87 \times 79 = 6873$) or the closet smaller decade (rounding-down) (e.g. doing $400 + 200 = 600$ to estimate $412 + 243 = 655$). If a set of numbers can be easily “fitted together” the compatible number strategy is a good estimation option. When using this strategy, it is best to view all numbers involved in a problem and find those that can form pairs, such as a hundred. For example, one can pair 26 and 81, 56 and 48, and 33 and 75 to form three hundreds to estimate the sum of the six numbers ($26 + 56 + 33 + 81 + 48 + 75 = 319$). This strategy essentially involves combining flexible rounding with experience. The front-end strategy focuses on the “front end”, or left-most digit, of a number, and is particularly useful in estimating solutions to addition problems. An example of using this strategy is calculating $1 + 5 + 3 + 4 = 13$ to estimate the sum of 1.02, 5.53, 3.14, and 4.33 (the sum is 14.02). The clustering strategy can be employed when a group of numbers “cluster around” a common value, such as when 68 and 72 cluster around the number “70”. For example, to estimate the sum of the five numbers, 78123, 80258, 80301, 79887, and 81926, one can observe that all these numbers are “about” 80000, and then simply multiply this figure by five, yielding a figure of 400000, which is very close to the actual solution of 399865.

Goodman (1991) proposed explicitly teaching estimation skills and strategies to students, and stressing mathematical concepts and properties related to computational estimation. However, those estimation skills and strategies are difficult to address in daily math lessons. Unfortunately, the failure of schools to properly teach these skills (Threadgill-Sowder, 1984) has led to students having poor estimation skills (Case & Threadgill-Sowder, 1990; Dowker, 2003; National Council of Teachers of Mathematics, 2000). Elementary mathematics education in Taiwan also suffers from this problem. According to the General Guidelines for the Grades 1-9 Mathematics Curriculum for Elementary and Junior High School Education (Ministry of Education, 2006), estimation is introduced from grade 3 onwards in elementary education in Taiwan. Estimation is included in the curriculum to help students develop estimation skills they can apply in problem solving. However, a lack of clear guidelines means teachers easily misunderstand the real objectives of estimation instruction and have their students learn estimation by operating calculator (Ong, 2005). Furthermore, teachers teach their students a simple rounding strategy rather than providing them with a complete introduction to and training in various estimation strategies. The instruction situation thus reflects the observation of Reys and Bestgen (1981) and Golden (1998), that estimation lessons typically teach rounding. In Taiwan, students are asked to provide a “unique” estimate by following specific directions, including “round up the answer to hundreds place” or “round down the product to the hundredths place”. The lack of practice in essential estimation skills (e.g. flexible thinking, decision-making, answer adjustment, and filtering out of non-sensible answers) has led to failure to develop student sensitivity and flexibility, both of which are crucial to effective estimation. Consequently, students tend to calculate the exact answer first and then “estimate” based on that answer, confusing them regarding the point of learning “estimation”.

According to the estimation process, a better way to help students develop estimation skills would be to view estimation abilities as a thread to be woven into instruction on estimation, computation, and problem solving. Reys (1986) proposed that the optimal method of developing estimation skills should combine three phases, namely instruction, practice, and testing (making it necessary assess the appropriateness of the selected strategies and the estimation results), which like problem-solving techniques are developed by careful instruction, discussion, and strategy use. Trafton (1988) also proposed that teachers should carefully motivate students, monitor student thinking, clarify study objectives, and be sensitive to the pace of instruction and estimation precision. Trafton (1988), van de Walle (2005), and Case and Threadgill-Sowder (1990) noted that teachers should employ appropriate examples to help students learn different estimation strategies and then employ problems to stimulate students to select an appropriate problem solving strategy via group discussion and reciprocal learning. Trafton (1988), van de Walle (2005), and Case and Threadgill-Sowder (1990) describe this approach to instruction as a problem-based learning (PBL) approach.

In PBL, students work in small collaborative groups and learn the knowledge required for problem solving. The teacher thus becomes a facilitator to guide student learning rather than a repository of knowledge (Hmelo-Silver, 2004). Based on the survey of Norman and Schmidt (1992) and the research of Hmelo-Silver (2004), PBL is identified as offering several potential advantages for student learning: (1) learning in PBL format may initially reduce levels of learning but over periods of up to several years may foster increased knowledge retention; (2) PBL curricula helps students develop flexible knowledge and effective problem-solving skills and enhances the transfer of both concepts and knowledge to new problems; (3) PBL achieves a sustained improvement in self-directed learning (SDL) skills; (4) PBL increases intrinsic interest in the subject matter being taught; and (5) PBL helps students develop effective collaboration skills. Although plenty of evidence exists supporting PBL in students learning, much of the research on PBL has been limited to higher education, mainly in medical schools. Few studies on PBL have considered K-12 education (Hmelo-Silver, 2004). Moreover, the majority of the research on PBL focuses on knowledge construction, problem solving, and SDL, and few works have focused on motivation and collaboration. Hmelo-Silver (2004) argues that PBL should be modified to support PBL for younger learners, particularly the SDL process, which may prove particularly difficult for those who tend to have difficulty applying metacognitive strategies.

The requirement of tailed PBL argued by Hmelo-Silver resembles the claim of Goodman regarding the design of teaching scenario of computational estimation. Goodman (1991) stated that further research on estimation should focus on the estimation instruction scenario, and moreover should clarify when specific estimation strategies should introduced, as well as what activities, materials, and teaching strategies are best suited to helping students acquire estimation skills. Although the importance of acquiring skills in computational estimation is widely recognized, few works have examined how to implement a problem-based instruction scenario to provide instruction in estimation skills. Furthermore, most research on computational estimation has focused on four topics: relationships between estimation abilities and other abilities (Goodman, 1991; Levine, 1982); identification of estimation strategies (Lemaire & Lecacheur, 2002; Threadgill-Sowder, 1984); error patterns (Reys, 1984); and development of numerical estimation (Case & Threadgill-Sowder, 1990; Siegler & Booth, 2004; Booth & Siegler, 2006). Few studies have applied collaborative learning to PBL to help students develop essential computational estimation skills. This situation may result from the difficulty of effectively implementing group discussion or reciprocal learning in traditional elementary classes. According to Lan, Sung, and Chang (2007; 2009), owing to the lack of target abilities, discussions (among the whole class or in small groups) tend to be dominated by students with stronger target abilities. To deal with the above problems associated with traditional cooperative learning activities, numerous researchers (such as, Lan et al., 2007; 2009; Huang, Kuo, Lin, & Cheng, 2008; Hwang, Yang, Tsai, & Yang, 2009) argued that mobile technology is not only a feasible means of resolving the above obstacles to effective group discussion and reciprocal learning, but can also help teachers observe and track student thinking because of its unique characteristics, including portability, social interactivity, connectivity, individuality, and immediacy.

Considering the limited number of investigations applying cooperative PBL to train elementary students in computational skills, and the very limited number of studies examining the effects of using mobile technology in such training settings, this study implemented a problem-based estimation instruction scenario and assessed effects on training student estimation skills, both with and without mobile technology involvement.

Method

Participants

Because of the limited quantity of mobile devices available (14 Tablet PCs), the study sample comprised just 28 fourth graders randomly sampled from two classes (each containing 26 students) that were stratified sampled from an elementary school in Taipei. All participants were randomly assigned to two groups: the experimental group (problem-based estimation instruction with mobile devices) and the control group (problem-based estimation instruction without mobile devices), each of which thus contained 14 students. The students were then grouped into several small estimation groups, each comprising three or four students. Consequently, each group comprised four small estimation groups, two of three students and two of four students. According to the General Guidelines of the Grades 1-9 Mathematics Curriculum for Elementary and Junior High School Education (Ministry of Education of the Republic of China, 2006), estimation is included in daily math classes, being introduced from grade 3 at the elementary level. Therefore, at least according to the curriculum, all participants had three semesters of exposure to estimation skills via their math class.

Design

This study adopted a mixed research approach, simultaneously gathering both qualitative and quantitative data. In terms of qualitative data, two observers recorded the treatment activities using two digital video cameras (each camera focused on one of the small estimating groups). After completing each unit, the two observers compared their observation results. Furthermore, following treatment the observers reviewed the videotapes of the group estimating process, focusing their observations on the following: (a) individual estimation, (b) self-reflection regarding their estimations, and (c) group cooperation to solve real world estimation problems.

For quantitative data, this study adopted an experimental design. Twenty-eight students were randomly assigned to two groups as detailed above (Participants section). All participants were administered a computational estimation test before and soon after treatment. Student test scores were gathered and analyzed to check whether the two groups performed or progressed differently in terms of computational estimation skills. Furthermore, to understand whether students selected appropriate estimation strategies or problem solving; following the post-test all the students were asked to write down the strategies they used to solve a real world problem which is described in the section on the assessment instruments.

Instruments

Estimation instruction materials

The estimation instruction materials included four units teaching the following estimation strategies: front-end, clustering, rounding, compatible numbers, special numbers, and initial estimate adjustment and refinement. Each teaching unit includes three stages: strategy introduction, individual practice, and group cooperation. Additionally, each strategy was introduced via examples and real world problems. Furthermore, the strategies were delivered via discussion (whole class and small group) and problem-solving activities, as detailed in the section on procedure. Appendix A presents an example of a teaching unit. The corresponding learning activities are detailed in the "Learning scenario" section. Additionally, each teaching unit was taught via two 40-minute periods each week, meaning the four units were taught over 4 weeks.

Assessment instruments

Assessment of the Computational Estimation Abilities of Elementary School Students (CEA Assessment). The CEA assessment was developed by Liu and Lin (1995), and is standardized and specifically designed for Taiwanese elementary school students. The instrument includes a three part computational estimation, involving a total of 36 items, including: order of magnitude estimation, reference number estimation, and open ended estimation. The measurement instrument used in this study comprised the first 24 items of the CEA assessment, *i.e.* 12 items involving order of magnitude estimation (e.g. identifying which option is closest to the answer of 9678-325: 80, 800, 8000, or 80000) and 12 items involving reference number estimation (e.g. estimating whether the result of 47 × 32 is bigger than 2000, bigger than 1200, smaller than 1000, or smaller than 800). To prevent students from performing actual calculations, each item was presented for only 15 seconds, after which students were asked to write down their estimates on a small answer sheet, as shown in Figure 1.The scoring mechanism gave each student one point for each correct estimation.

Figure 1. The answer sheet used in the CEA assessment

Assessment of applying estimation strategies (AES assessment). This test assessed student metacognition knowledge regarding selecting and using appropriate estimating strategies to solve a real world problem. The students were shown the item for only 15 seconds and were then asked to write down their estimation strategies. The problem is detailed below.

> *Some guests are visiting your house this Saturday. Mom has gone to the supermarket to shop for the guests, and is now waiting at the check out, with just one other shopper ahead of her in the queue. At this point she suddenly realizes that she has only 600 NT dollars, and worse still has forgotten her credit cards. She must quickly decide whether she has enough money to pay for her shopping.*
> *The prices for the individual items are as follows: 39, 23, 151, 48, 62, and 82 (NTD)*

Learning Environment

The Group Scribbles platform. Group Scribbles (GS), developed by SRI international (SRI International, 2007), enables collaborative generation, collection and aggregation of ideas via a shared space based on individual effort and social sharing of notes in graphics and text form. GS supports the coordination of different processors and operates on various client computing devices, including laptops, Tablet PCs, and PDAs. In this study, each student in experimental group was provided a TravelMate C110 Convertible Tablet PC to write down their estimation results. The main GS operation interface takes the form of a three-paned window, as illustrated in Figure 2. Meanwhile, the lower pane is the user's personal work area, or "private area", which contains a virtual pad of fresh "e-sticky notes" on which the student can record their estimate. The "tool bar," shown to the right of the private area, enables users to select the note color and choose whether to input data by drawing or typing. An e-sticky note can be made visible to others by dragging it into the "public area" in the middle upper pane, which is synchronized across all devices. Executing a reverse drag makes the item private again. Users may interact with public e-sticky notes in various ways, including browsing, repositioning them, or moving them from the public area and into their private space. The right upper pane is used for group work, which supports student idea sharing in small groups. An e-sticky note can be invisible to others but group members can see it by dragging it into the group area work. Additionally, the left side of Figure 2 contains a frame called the "control area", which is only visible to the teacher, and which lists all the student numbers and helps the teacher to group students into small cooperative groups.

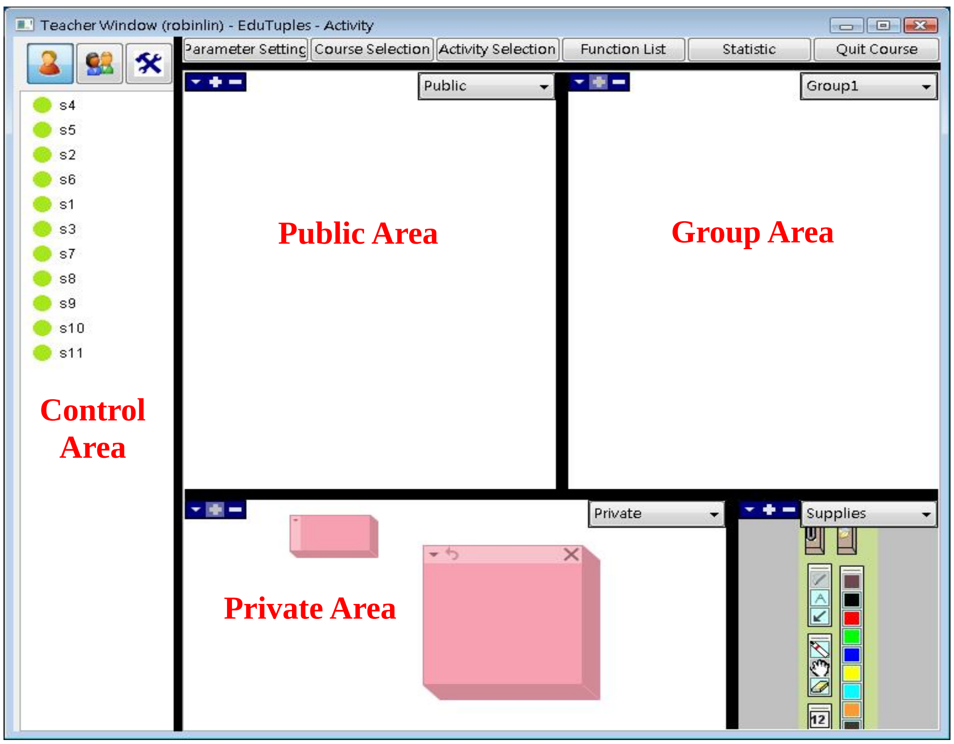


Figure 2. GS operation interface

Learning scenario. To facilitate individual and cooperative learning in PBL for elementary students and cultivate their computational estimation skills, the learning scenario for each teaching unit involves three stages: strategy introduction, individual practice, and group cooperation. During the first stage, strategy introduction, a power-point file presenting a problem-based story is projected onto the screen accompanied by several embedded problems to train students in essential computational estimation skills. The learning activities during this stage focus on class discussion, and the teacher leads the class in identifying the problem and devising the most appropriate estimation strategy. Appendix A. (a) illustrates a fragment of a story line used to introduce and teach students the compatible number and rounding strategies. During this stage, the teacher encourages each student to write down their estimates and drag the e-sticky note into the public area for sharing with the whole class, as illustrated in Figure 3. Changes in the estimation result are permitted within the time limit. Therefore, each student can change their mind if they think someone else has a better answer than theirs. Additionally, to confirm that students understand why and how to choose an appropriate strategy, the teacher asks students why they selected their preferred strategies and how they obtained their estimates.

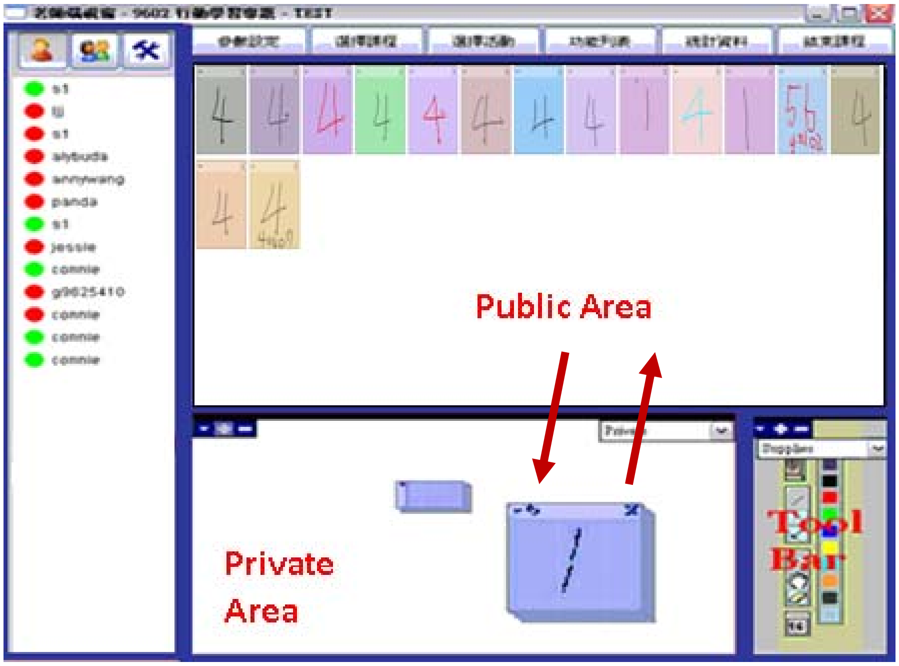


Figure 3. Example of individual estimation results from the strategy introduction stage presented for discussion by the whole class

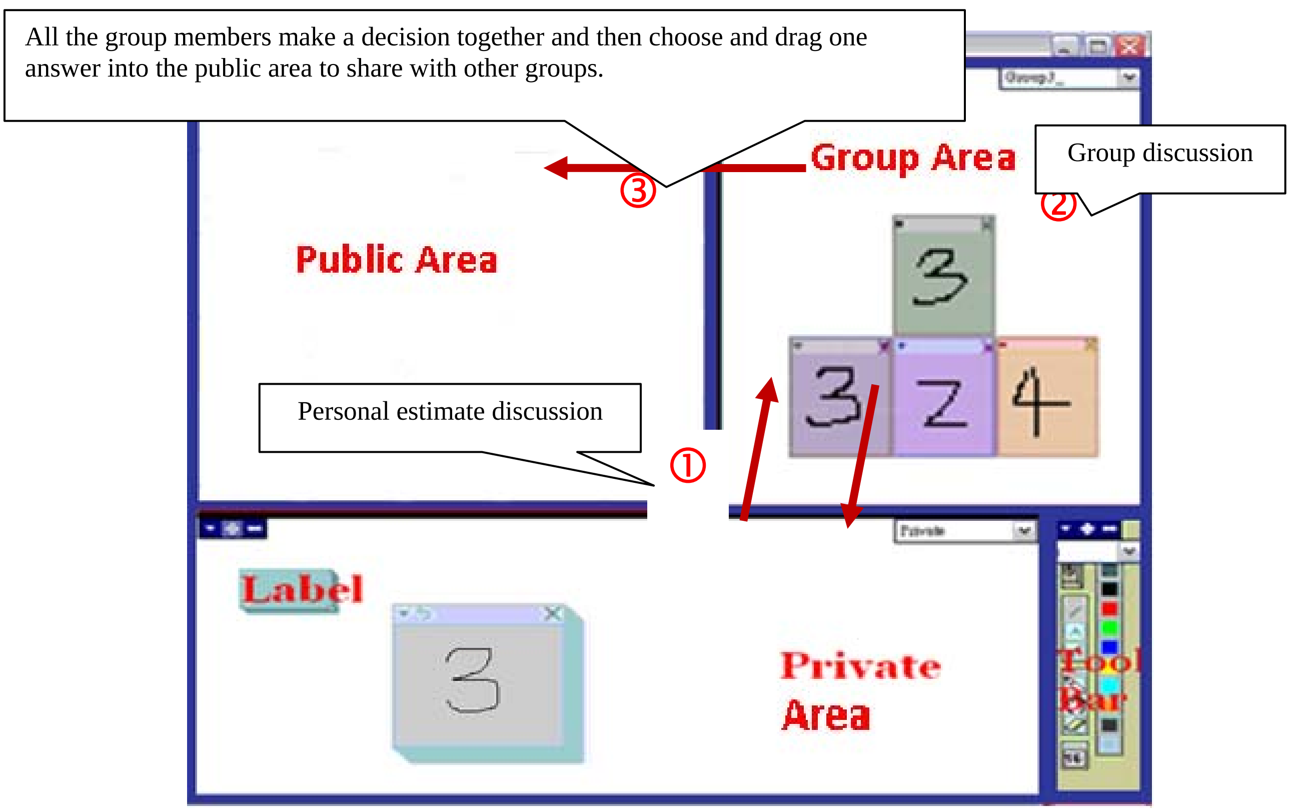


Figure 4. Example of group cooperation -- group discussion

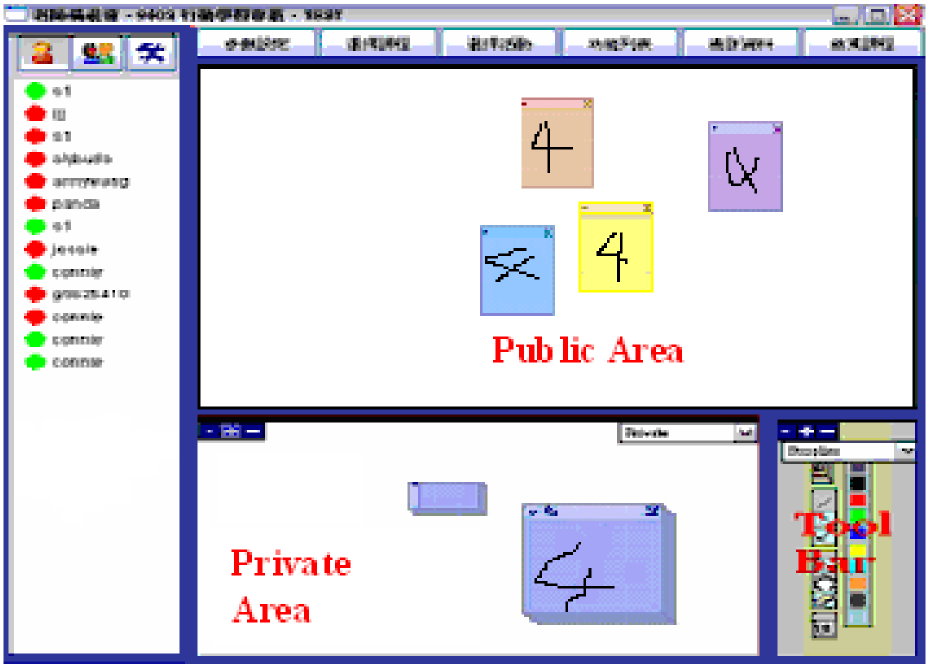


Figure 5. Example of the final results of group cooperation

After all the strategies included in a unit are introduced to students, a series of similar estimation problems is employed to train each individual to master the taught estimation strategies, namely, the individual practice stage. During this stage, the estimation problems are projected on the screen individually, and each student is asked to write down their answer on an e-sticky note in the private area, before dragging it to the public area (as shown in Figure 3). When the time is up, the teacher leads the whole class in discussing and deciding on the correct answer. Appendix A. (b) shows the practice problems used to help the students master the compatible number and rounding strategies. Finally, during the group cooperation stage, a challenge problem is projected on the screen, and all groups are asked to conduct intra-group discussion, select the most appropriate estimation strategy and also provide the correct estimate. Each group member first writes an estimate on their e-sticky note in the private area, and then drags the note to the group area, as shown in Figure 4. While conducting group discussions, a group member can change or insist their estimate depending whether they agree with the opinions of others. As soon as they reach agreement, group members drag their final estimate result to the public area for sharing with other groups, as shown in Figure 5. Finally, the teacher leads the class to confirm the estimate results and also encourages each group to share their reasons for strategy selection. Appendix A. (c) illustrates an example of a challenge problem used during the group cooperation stage.

Figure 6. Sticky notes containing student estimates (the control group)

Sticky notes

A stack of sticky notes was provided to each of the estimation groups within the control group. While practicing estimation strategies or solving estimation problems, the students in the control group used these sticky notes to record their ideas or estimation results, thus facilitating sharing with others (as shown in Figure 6).

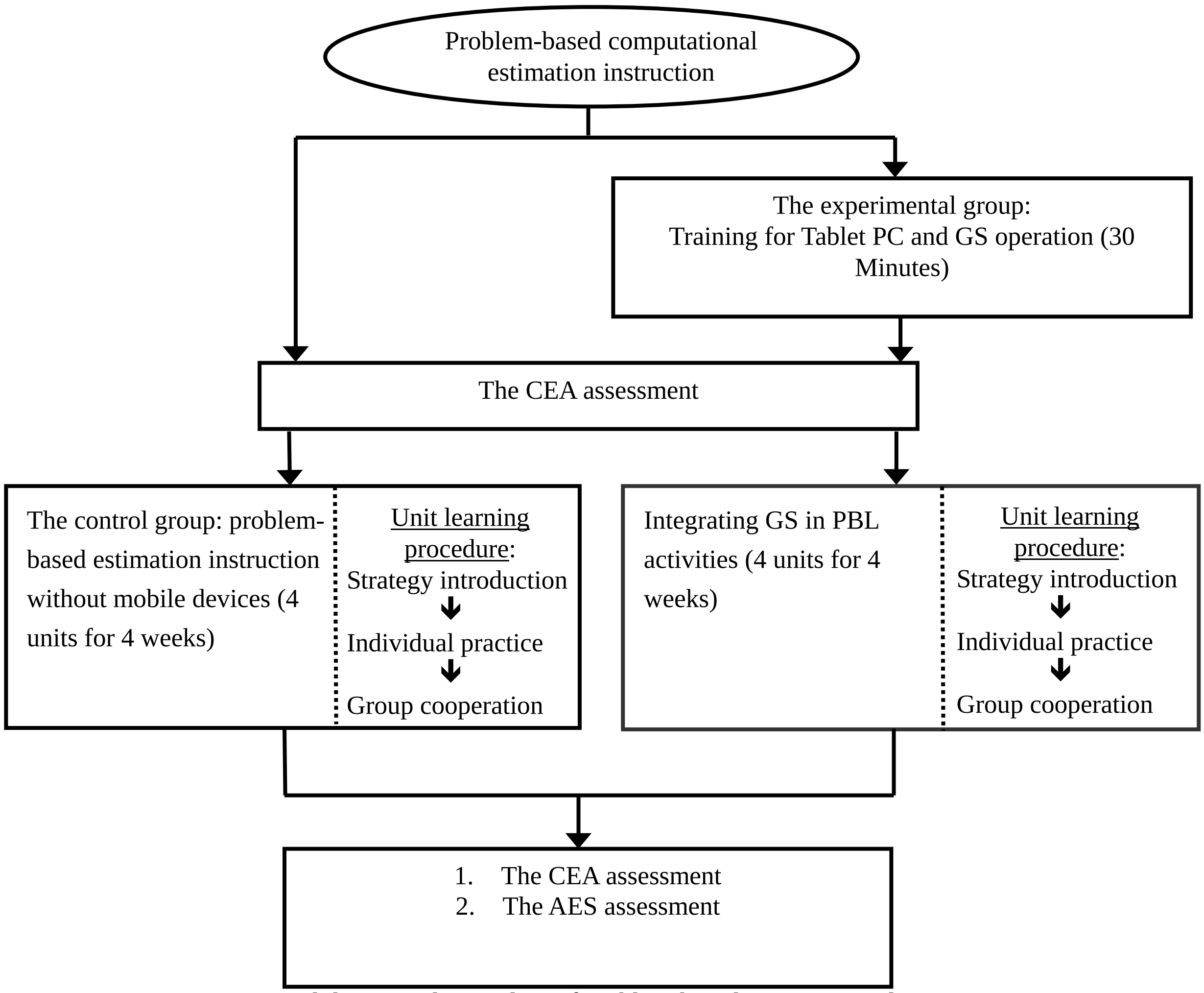


Figure 7. Experimental design and procedure of problem-based computational estimation instruction

Procedure

Figure 7 summarizes the experimental procedure. Before treatment, as shown in Figure 7, the experimental group received 30-minutes of training on using the Tablet PC and GS system, after which all students were administered a pre-test involving CEA assessment. Then four units, each lasting 80 minutes, were taught during the 4-week experiment. Each unit comprised three instruction stages: strategy introduction, individual practice, and group cooperation. Each student in the experimental group was equipped with a Tablet PC. The students logged into GS, and the teacher then grouped them into small estimation groups of three or four members. During the strategy introduction stage, the three essential phases of teaching estimation proposed by Reys (1986) were embedded in various learning activities following a “personal estimation – whole class judgment” cycle. The teacher first presented a problem-based story to the class and led them in helping the main character in the story to solve problems. After the character encountered a computational estimation problem, the teacher asked each student to estimate the solution. Students wrote or typed their ideas or estimates on the e-sticky notes and dragged them to the public area for sharing with the whole class (see in Figure 3). Subsequently, the class discussed various estimation strategies and selected the best. Once the class reached agreement, the teacher continued the story and led the class to solve another problem to help the students practice estimation and learn the target estimation skills. The story was followed by a series of practice problems, comprising the individual practice stage. All students were asked to solve the estimation problems and shared and discussed their solutions with the class.

The group cooperation stage followed soon after the individual practice stage, during which students learned the target skills contained in a single unit. During this stage, the teacher presented a real world problem and asked students to cooperate to develop a suitable solution. Each member of each small estimation group wrote down their estimate on an e-sticky note and then dragged it to the group area to show their groupmates (see Figure 4). The group members then discussed and tried to reach agreement on a strategy to be shown in the public area, as shown in Figure 5. The teacher then led the class in assessing the estimates of the various groups.

In contrast with the experimental group, the students in the control group received estimation instruction materials and instruction flow identical to those described above, but simply wrote down their estimates on the sticky notes, as shown in Figure 6.

Upon completion of the treatment, all 28 subjects were re-administered a post-test dealing with the CEA assessment. The students were also administered the AES assessment to solve a real world problem, with the strategies they used to do this being recorded.

Results

Comparison of computational estimation skills

Results of the Assessment of Computational Estimation Ability

The computational estimation ability of all students was tested twice (both before and after treatment). The alpha was set to .05.

Table 1 lists the means and standard deviations for the computational estimation ability scores. Two-way (test × group) analysis of variance reveals that the group is insignificant ($F(1,52) = 0.45$, $p > .05$), meaning the scores do not differ between the two groups. The test is significant ($F(1,52) = 12.20$, $p < .05$), indicating a difference between the pre- and post-test scores. The interaction between group and test is insignificant ($F(1,52) = 0.01$, $p > .05$), meaning no level varying differences exist. Additionally, further tests of the progress between the pre- and post-test conditions for the two groups showed that the experimental and control groups both achieved significant improvement in their computational estimation skills ($F(1,52) = 5.84$, $p < .05$ for the experimental group; and $F(1,52) = 6.36$, $p < .05$ for the control group). However, although the standard deviations of the two groups reduced between the pre- and post-test, the more condensed score distribution for the performance of the experimental group demonstrated a smaller difference in computational estimation abilities between the students of the experimental group.

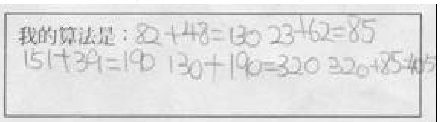
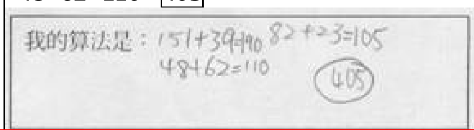
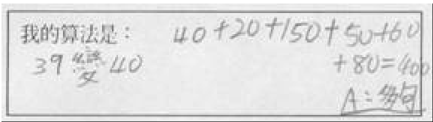
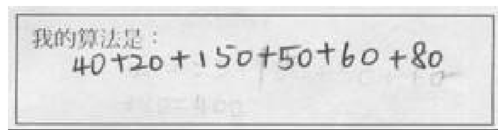
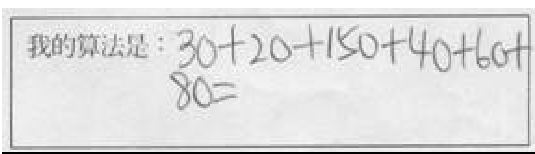
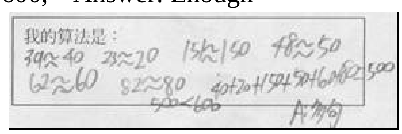
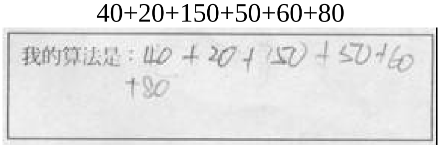
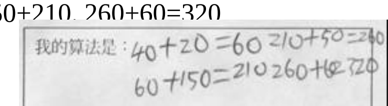
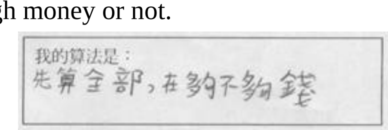
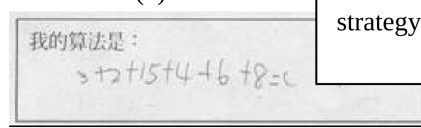
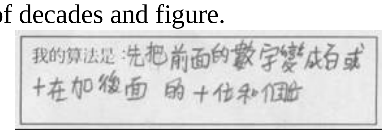
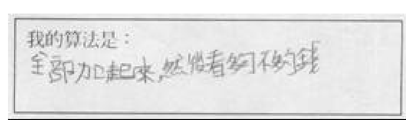
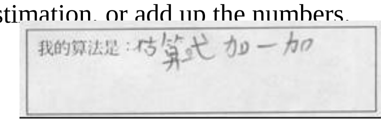
Table 1. Means and standard deviations for the scores of ability of computational estimation

Test	**Experimental group (N =14)**		**Control group (N = 14)**	
	M	***SD***	***M***	***SD***
Pretest	12.86	4.49	12.14	3.53
Post-test	16.14	3.18	15.57	3.01

Comparison of Metacognition Knowledge of Computational Estimation Abilities

Besides the CEA assessment, all participants were asked to solve a real world problem following the post-test. Eight students (students 1 to 8) in the experimental group applied estimation strategies to solve the problem, as shown in Figure 8, where each cell presents the original estimates of students (the scanned rectangle) and the corresponding English translation.

Compatible numbers strategy

No.	Estimation approach	No.	Estimation approach
1	My approach: 82+48=130, 23+62=85 151+39=190, 130+190=320, 320+85=405 	2	My approach: 151+39=190, 82+23=105 48+62=110 405 
3	My approach: 39 change to 40, 40+20+150+50+60+80=400 Answer: Enough 	4	My approach: 40+20+150+50+60+80 
5	My approach: 30+20+150+40+60+80= 	6	My approach: 39≈40, 23≈20, 151≈150, 48≈50 62≈60, 82≈80, 40+20+150+50+60+180=500 500＜600, Answer: Enough 
7	My approach: 40+20+150+50+60+80 	8	My approach: 40+20=60, 210+50=260 60+150+210, 260+60=320 
9	My approach: I will count the all parts, and estimate if there is enough money or not. 	10	My approach: 3+2+15+4+6+8=( ) 
11	My approach: First, I will change the numbers in front to hundreds, or decade, then I will add them with the numbers of decades and figure. 	12	My approach: I will add up all the numbers, and then see if there is enough money or not. 
13	My approach: Use estimation, or add up the numbers. 	14	My approach: Use estimation. 

Rounding strategy

Figure 8. Student estimation methods in the experimental group

Seemly-rounding strategy

Seemly rounding strategy

Seemingly rounding strategy

No.	Estimation approach	No.	Estimation approach
1	My approach: 39 is calculated to 40, 23 is calculated to 25, 151 is calculated to 150, 48 is calculated to 50, 82 is calculated to 80 Add the whole parts, and use estimation. 我的算法是：39算40，23算25，151算150，48算50，62算60，83算80，加起來看看，用估算的方法	2	My approach: 40 45 375＜600 20 60 150 80 我的算法是：40 45 20 60 150 80 375<600
3	My approach: 48 is changed to 50, 151 is changed to 150 39=40 我的算法是：48變50 151變成150 39=40	4	My approach: Use estimation, and add up the numbers which are closed to each other. 我的算法是：用估算，將比較接近的數加起來。
	ch: ole parts, and see if the money is not. 我的算法是：把全部加起來，看錢夠不夠	6	My approach: 100+30+20+80+40+60=320 我的算法是：100 30 50 80 +40 320
7	My approach: Add everything together. 我的算法是：全部加在一起	8	My approach: First, I will try the simple approach, and then I will try the harder one. 我的算法是：先把簡單的算法算一算，再算難的算法。
9	My approach: Add everything together. 我的算法是：全部都加起來	10	My approach: 9+3+1+8+2+2=25 30+20+150+40+60=300 300+25=325, Answer: Enough 我的算法是：9+3+1+8+2+2=25 30+20+150+40+60=300 300+25=325 A:夠
11	My approach: Use estimation. 我的算法是：用估算的方式	12	My approach: Use addition. 我的算法是：用加的
13	My approach: 3+2+4+6+8=23 4+3+51+8+2+2=75 我的算法是：3+2+4+6+8=23 4+3+51+8+2+2=75	14	My approach: 我的算法是：

Figure 9. Student estimation methods in the control group

Notably, students 1 and 2 used a compatible numbers strategy, while students 3 to 8 employed a rounding strategy. In contrast, only two students, students 1 and 3, from the control group seemed to clearly describe their estimation strategy. However, the judgment of student 3 regarding the estimation strategy was based only on the processing of the three numbers (48, 151, and 39), but whether student 3 knew how to manipulate the other three numbers (23, 62, and 82) is uncertain. Furthermore, the strategy used by student 1 appeared a rounding strategy, but this was not entirely correct because of the way the number 23 was rounded. Student 1 "rounding" 23 to 25 is not consistent with the rounding strategy as defined in this study. The rounding strategy involves rounding a number to the closet larger decade (round-up) or the closet smaller decade (round-down). The strategy adopted by student 2 suffered the same problem as that of student 1. Furthermore, the number processing result of the estimate of students 2 was unclear because no relationship existed between the substituted and original numbers. Figure 9 illustrates the estimation strategy based on the results of the control group.

By comparing estimation strategies used in the two groups (the experimental and control groups), chi-square analysis, this study used Yate's correction of continuity to correct the chi-square testing results owing to the small sample size, and demonstrated significant differences between the two groups ($\chi_{(1,1)} = 3.89$, $p < .05$). This analysis demonstrates it is significantly more common to apply an estimation strategy to solve a real world problem in the experimental group than in the control group. The combination of the problem-based instruction approach with mobile device thus appears to help students cultivate their ability to apply estimation strategy.

Results of in-class observation

Two observers observed the processes of personal estimation and intra-group cooperation, focusing on individual estimation, self-reflection on individual estimations, and group cooperation. Students from both groups were actively involved in most of the estimation activities. However, students from the control group rarely modified their estimates during the practicing stage, while students from the experimental group did. Students from the latter group dragged back their e-sticky notes and re-dragged modified estimates (on another e-sticky note) to the shared area after comparing their estimates with those of others. During the group cooperation stage, the control group frequently encountered a situation in which certain students dominated the discussion and estimation strategy selection. Most small estimating groups there were two or three students who typically just listened to their groupmates explaining why they chose a particular problem solving strategy, and then reached agreement with minimal discussion. Unlike those in the control group, students in the experimental group were more eager to display their e-sticky notes and express their ideas, and sometimes they were unable to reach agreement.

Discussion

Computation estimation skills are important to student mathematical skills. However, computational estimation skills are difficult to teach in math class. Most teachers in Taiwan ask students to calculate a precise answer first, and then make an "estimate". Owing to a lack of training in flexible thinking, most students tend to calculate answers to problems without considering the characteristics of the numbers or how they are related. Although researchers have proposed various approaches, including regular instruction in problem solving, to computational estimation instruction, few studies have attempted to implement such problem-based approaches in daily instruction on computational estimation, and nor have many studies assessed its effects on student computational estimation abilities. Based on the comparison of the scores of the pre- and post- tests for CEA assessment, the three-staged problem-based estimation instruction scenario (strategy introduction, individual practice, and group cooperation) proposed here was found a useful means of developing student estimation skills. Given regular and explicit introduction and practice, students belonging to the two groups achieved significant progress in computational estimation skills. This finding is consistent with the arguments of Trafton (1988) and van de Walle (2005). However, based on further comparison of the standard deviations between the two tests (pre- and post-test) for the two groups, involving GS in a problem-based estimation instruction scenario promotes estimation skill development for most students in the experimental group rather than only benefiting those with high abilities.

Furthermore, according to Lemaire and Lecacheur (2002), younger learners (e.g., six graders and fourth graders) favored the round-down over the round-up strategy, and almost always selected the former, probably because the rounding processes were easier to execute. Lemaire and Lecacheur inferred two possible causes for this phenomenon: first, the round-up strategy requires calculating the differences between unit digits and the closest larger decades; second, round-up processes require more working memory for storing the decade digits, which were not the original digits from the problems. However, regarding student application of the rounding strategy in this study, students were found to flexibly use both rounding strategies, up and down. For example, student 6 of the experimental group flexibly used the rounding strategy to process the numbers according to their characteristics (e.g., rounding up 39 to 40, rounding down 23 to 20, rounding down 151 to 150, rounding up 48 to 50, rounding down 62 to 60, and rounding down 82 to 80). This finding further shows that the proposed instruction approach appeared able to help students overcome the mental effort required to implement the round-up strategy.

Regarding the results of AES assessment, this study found that significantly more students in the experimental group than the control group could choose appropriate strategies for solving real world problems. This result appeared that the mobile-device-supported problem-based computational estimation instruction helped students develop metacognition knowledge of estimation strategies. This outcome may result from regular student self-reflection regarding the estimates, as well as discussion and cooperation in each small estimation group in the experimental group, as confirmed via in-class observation. Although the students in the control group were also grouped in small estimation groups, encouraged to cooperate and discuss the task together, and learned the target estimation skills by performing problem-solving activities, students working in small groups tended to perform "dominated learning". In the control group, students with higher target abilities frequently dominated the "cooperative learning", while students with lower target abilities almost always listened to and agreed with the estimation approach, or with results provided by their group mates. This situation resembles the series of studies conducted by Lan, Sung, and Chang (2007 & 2009). The lack of self-reflection and self-estimation appears to prevent students from developing metacognition knowledge of estimation strategies, even when their performance as measured by the CEA assessment improved.

Conclusion

This study proposed a three-stage problem-based estimation scenario involving the implementation of a problem-solving based approach to estimation instruction. From the analytical results, the three-stage problem-based instruction scenario appears to help students develop their estimation skills. Additionally, blending this scenario in a mobile-device-supported cooperative learning environment helped students cooperate and thus appears to benefit not only the development of estimation skills in elementary students, but also that of metacognition knowledge of estimation strategies. Furthermore, this study demonstrates that the change of student computational habits, from directly performing calculations without first discovering the characteristics of the numbers involved and their interrelationships to applying flexible thinking to select an appropriate estimation strategy for solving an authentic problem, is critical to the development of numerical sense and manipulation ability. The findings of this study have increased understanding of the impact of mobile technology on student cooperative learning and estimation skill development. The problem-based instruction scenario proposed in this study provides a potential answer to the question of Goodman (1991) regarding when to introduce various estimation strategies to students, as well as the activities that offer potential to help students acquire estimation skills. The proposed scenario also provides math teachers with an easy to implement approach for teaching computational estimation. However, hardware limitations meant that the sample size in this study was small. Future studies should use larger sample sizes and consider students from different grade levels. Additionally, according to Norman and Schmidt (1992) and Hmelo-Silver (2004), the effects of the proposed approach might last up to several years, increasing estimation knowledge intention. However, because of the short treatment period, this study lacks evidence clarifying the existence of such an effect. Therefore, the long-term effect of integrating mobile technology in PBL activities related to estimation instruction deserves attention in future.

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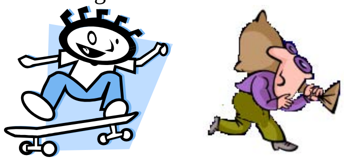
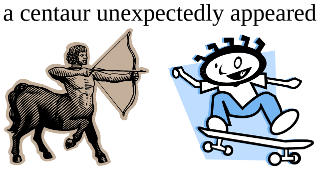
Appendix A. Sample unit used to train students in the compatible number and rounding strategies

(a) Strategy introduction: a fragment of a story line used to teach students the strategies of compatible number and rounding.

① Xiao-Xin found that his dog, Xiao-Bai, was kidnapped and caged in a kindergarten by a bad guy. Therefore, he hurried to the kindergarten to rescue Xiao-Bai. 	② Suddenly, a turtle goblin appeared, stopped Xiao-Xin, and said "Answer my question or you will never be able to save your dog. Remember, you have only 10 seconds to give the answer. Ha! Ha! Ha!" 
③Then, the turtle goblin waved her hands, and a math problem appeared in the air. The problem was "999+9999 =?"	④ Xiao-Xin gave the correct answer, and the turtle goblin let him go despite being very unhappy with the outcome.
⑤ Xiao-Xin continued on his way to the kindergarten, but then a centaur unexpectedly appeared and blocked Xiao-Xin's path. 	⑥ The centaur also asked Xiao-Xin to answer his question in 10 seconds. To be allowed to continue his journey Xiao-Xin had to solve the following problem. The question was: Which is the correct answer to "198-19-78"? 198-19-78= ? (1)111 (3)110 (2)101 (4)119
⑦ …	

(b) Individual practice: examples of the practice problems used to help each student master the strategies of compatible number and rounding (with students having 10 seconds to solve each problem).

①

6+12+74= ?
(1)92
(2)144
(3)82
(4)94

②

37-18-17= ?
(1)19
(2)36
(3)2
(4)12

③

99+99= ?
(1)199
(2)198
(3)200
(4)188

④

67+99= ?
(1)100
(2)166
(3)190
(4)200

(c) Group cooperation: an example of a challenge problem used in group cooperative learning activities.

挑戰題

媽媽到迪化街辦年貨，香菇一斤603元，瓜子一斤99元，橘子一斤9元，糖果一斤8元，媽媽身上有 1500元，如果買了半斤香菇、6斤瓜子、12斤橘子和19斤糖果，請問媽媽的錢夠不夠？

Mom is going to Di-Hua street to do Chinese New Year shopping. The prices for the goods she wants are as the follows: dried mushrooms, 603 NTD per Kg; sunflower seeds, 99 NTD per Kg; oranges, 9 NTD per Kg; candy, 8 NTD per Kg. Mom has NTD $1500. She wants 0.5 Kg of dried mushrooms, 6 Kg of sunflower seeds, 12 Kg of oranges, and 19 Kg of candy. Does Mom have enough money for her shopping? (15 seconds)