Preservice Teachers' Perceptions of Applets for Solving Mathematical Problems: Need, Difficulties and Functions

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ABSTRACT

This research describes preservice teachers' perceptions of the use of applets in solving mathematical problems. It examines such perceptions regarding the role and functions that applets have in the problem solving context, the difficulties they encounter when using applets in this context and the necessity or need of their use. I attempt to categorize the perceived need in terms of the types of solution that were used by the participants and the difficulties that they met during solving mathematical problems. To analyze the participants' solutions, their difficulties and need for applets I use the content analysis method which enables to determine the presence of certain words or concepts within the text and the relation between these words or concepts. To discuss the participants' perception of the need for applets I used the activity theory frame which enables to analyze the participants' opinions by looking at their operations, actions and activity, and, at the same time, looking at their goals and motives. Though most of the participants thought that mathematical problems could be solved without applets, they emphasized the role of applets as fostering, facilitating and clarifying mathematical problems' statement and solution. The participants pointed at applets as tools which learners enjoy working with, so they will be encouraged to solve mathematical problems using them. What influenced the participants' perception of the need to use applets for solving mathematical problems were their ability to solve problems using them, their ability to solve problems without them and the type of difficulties faced by them during the solving process.

Keywords

Applets, Mathematical problems, Preservice teachers, Perceptions, Activity theory

Introduction

Background

The use of the internet as an educational medium is now rapidly expanding (Liaw, 2004). Sinclair (2000) points that the internet is becoming an intriguing environment for mathematics education by combining a variety of modes that can be applied to learning. Engelbrecht and Harding (2005) say that there are an increasing number of mathematical sites that use applets to enhance their pages with animated figures and interactive illustrations. They point that these sites include good mathematical applets that are visually of great value and that can be fruitfully used as educational tools. Applets are programs that require a WWW browser or other application to run. Usually these tools represent scientific concepts algebraically and visually, so they enable learners to perceive different representations of the same concept. Examples of mathematical applets can be found at many educational sites, for example the National Council of Teachers of Mathematics' (NCTM) site or the Center of Educational Technology' (CET) site, where the first site includes applets that treat different mathematical topics, while the second site includes applets that treat two topics: linear and quadratic functions. Usually an applet treats a specific topic, for example number factors, decimal fractions, the circle area, etc. So to cover a whole subject one needs different applets, for example one needs to work with applets that treat the following topics to cover the linear function subject: input-output, rate of change, transformations, transformations of straight lines which pass through the origin, addition of linear functions, subtraction of linear functions, forms of linear functions, and functions defined on intervals. The CET's site includes an applet for each topic mentioned above, where the topics are presented in a problem solving context. I've been introducing this environment for preservice teachers for almost 8 years now. Some of them find difficulties working with applets to solve mathematical problems, while others don't find such difficulties, some don't consider applets to be needed in solving mathematical problems, while others are attracted to these new tools and find them helpful in solving the problems. I have always asked myself about the factors that make some learners find difficulties while working with applets while others don't do so, and about the factors that make some learners approve working with applets while others think applets are not needed. This research came to answer these questions.

Literature review

Bolyard and Moyer (2003) say that students in middle grades sometimes need extra help and scaffolding to make the transition from concrete tasks to abstract concepts. They suggest to use contextualized problems with multiple
representations that applets can provide as means to help perform this transition. Young (2006) provided a summary of the current literature on applets in mathematics education and described their benefits as found in various articles: (1) their availability on the internet and thus their free and ease of access (2) their focus on specific concepts (3) applets enable learners to do things that are not possible or easy with physical manipulatives, or pencil and paper (4) applets provide students with instantaneous and corrective feedback, so they fit inquiry-based learning and problem solving (5) applets provide multiple representations of a single concept at the same time; thus they can promote the transfer of knowledge from specific ideas to general knowledge (6) applets may be helpful for students with disabilities (7) applets increase motivation and attention in students as well as teachers.

The ability of applets to represent mathematical phenomena in multiple representations and to motivate students' learning of mathematics make them an appropriate tool for solving mathematical problems, especially word problems which students don’t look forward to solve them. It may be expected that the ability of applets to offer multiple representations can introduce different types of solution for mathematical problems, for example: dynamic solution (applet only solution), blended solution (applet and algebraic). These main aspects of applets encourage exploring their use for solving mathematical problems which are usually solved in algebraic methods.

Eason and Heath (2004) said that applets enhance mathematics learning and added that they helped immensely when first introducing solution methods for discrete dynamical systems. They pointed that students using applets dealing with discrete dynamical systems could manipulate the parameters of a given problem and thus obtained a system appropriate for it. Then they could plot and visualize the behavior of the system. Eason and Heath described how the learning of the students was enhanced by working with an applet's window which computed the numerical solution of the system and with an explanation window. All these options enabled students to explore deeper concepts behind the behavior of the discrete dynamical systems.

Gadanidis, Gadanidis, and Schindler (2003) found that the use of applets was one of several factors; typically not the major factor, affecting the pedagogical thinking of preservice teachers when planning mathematics lessons that use applets. They mentioned the following factors as affecting the preservice teachers' pedagogical thinking: the mathematics' topic, their pedagogical beliefs, personal mathematics' knowledge, comfort with technology, classroom management concerns, past mathematics learning and teaching experiences, and recent teacher's educational experiences.

Smetana (2003) described how two 8th grade students and one 7th grade students worked with applets to solve balance problems. First they solved balance problems in worksheets, and then in applets. The students found working with applets difficult because they had hard time using the program. They also preferred working with the problems in the worksheet over the problems in the applets. Smetana commented on her experience saying that the kind of applets that she used would probably benefit younger students who hadn't been introduced to balance problems yet.

Gadanidis (2002) reports case studies of three 5th and 6th grades teachers and one 10th grade teacher who used applets in their teaching of mathematics. The 5th grade teacher worked with students on maximum area problems, first without applets and then with applets, and was surprised how all students, not only the top ones, could solve maximum area problems without much difficulty. The 6th grade teachers worked with their students on the same problems, using applets from the beginning. They reported that many of their students didn't know what to expect, thought that working with applets was for fun and didn't know how to solve the maximum area problems using them. The 6th grade teachers concluded that they would better first familiarize the students with the problems and then ask them to solve them with applets. The 5th and 6th teachers said that working with applets made them aware of the relation between perimeter and area, which they hadn't been aware of before. The 10th grade teacher reported that the class who worked with applets gained significantly higher scores than the class who didn't. The teacher also pointed that working with applets has shifted his attention from teaching individual and isolated effects of coefficients of quadratic equations to holistic and dynamic exploration of relationships between quadratic equations and graphs.

Theoretical framework

Activity theory

Activity theory is a cultural-historical theory of activity which was initiated by a group of Russian psychologists in the 1920s and 1930s. This approach was led by Lev Vygotsky (1896-1934) and his colleagues A. N. Leon'tev and A. R. Luria (Engestrom, 2003).
The basic structure of an activity is represented in Figure 1.

![Figure 1: the basic structure of an activity](image)

In this structure, an activity is composed of a subject and an object, mediated by a tool, where the subject is a person or a group engaged in an activity, while an object is held by the subject and motivates the activity, giving it a specific direction. The mediation can occur through the use of different types of tools, material as well as mental, including culture, ways of thinking and language (Mappin et al. 1999). According to Leont'ev (1979), analysis, using activity theory, takes into account three levels: analyzing the activity and its motive, analyzing the action and its goal, and analyzing the operation and its conditions. Leont'ev describes the actions as the activity components that translate it into reality. Operations refer to how actions can be done under objective circumstances. The structure that Leont'ev suggests for an activity is represented in Figure 2.

![Figure 2: the hierarchical structure of the activity according to Leont'ev](image)

This structure will be used to explain the relations between the history of the participants, regarding their use of applets in learning mathematics, and their perceptions regarding the need for applets when solving mathematical problems.

Areas of ICT integration in learning

Wang and Woo (2007) say that, depending on the scope of content covered, ICT integration can happen in three areas: curriculum, topic and lesson. They further elaborate explaining that ICT integration into the curriculum requires ICT to support a more substantial amount of subject content which contains a number of topics in a specific discipline. ICT integration in a topic can be used to cover certain topics within a course. At the micro level, ICT is used to help explain specific knowledge units which can be covered within a single lesson. This structure of ICT integration will be used to categorize the functions perceived by the participants to be played by applets in mathematical problem solving.
Martínez-Santaolalla et al. (2005) describe three phases of learning that students should progress through in order to acquire true understanding of mathematics: action (involving physical and mental manipulations), abstraction, and reflection (when students consciously analyze their thinking). Martínez-Santaolalla et al. point that cycling through these phases time after time enables students to construct increasingly sophisticated mental models of the mathematical phenomena studied.

The participants in this research went through all the previously mentioned three phases of learning when they solved problems from the CET site.

**Rationale for the study**

Preservice teachers should be exposed to technological tools that would help them in their future teaching. They should first be exposed to these tools as learners, so they would appreciate them as tools that facilitate learning, and thus, when they come to teach, they will introduce them to their students as objects with which they can learn. Technological tools can have various functions in the mathematics classroom; one of which is to solve mathematical problems with them, which is an important aspect of the students' mathematical activity (NCTM, 2000). In this research, I want to examine the work of preservice teachers with applets to solve mathematical problems and how they perceive this work. This perception can point to whether and how they will use the applets as teaching tools. This research will also help us better know how to introduce applets as solving tools to preservice teachers.

Designers would benefit from this research because they will become more knowledgeable regarding the difficulties that users experience when working with applets. This would help them design more efficient applets that students have less difficulties when working with them.

Young (2006) says that very little formal research has been performed on the effectiveness of applets. This research is interested to explore the perceptions of preservice teachers regarding the functions, effectiveness, and benefits of applets, while solving mathematical problems. Doing so, this research attempts to permeate a little researched field.

**The research questions**

1. What are the roles and functions of applets that preservice teachers perceive when using them to solve mathematical problems?
2. What are the difficulties faced by preservice teachers when solving mathematical problems with applets?
3. What is the relation between the perceived need for applets to solve mathematical problems, and the types of solutions and difficulties faced during the solving process?

**The research method**

**The research setting and participants**

The research was conducted in two mathematics didactics classes of preservice teachers who learned in an academic college of education. The preservice teachers were in their second year of study to major in mathematics and computer education. These classes were held in two consecutive academic years 2006-2007 and 2007-2008. The first class included 27 preservice teachers, while the second included 28 preservice teachers. The mean age of the participants was 20.7 years with standard deviation of 1.3. The didactics course introduces the preservice teachers to various technological methods of teaching mathematics: applets, video, spreadsheets and wiki. The didactics course follows a geometry course in which the preservice teachers are introduced to and work with applets.

**The assignments**

The participants solved two assignments with applets. The first assignment was a task in the geometry class, where the participants were required to solve a geometry problem using an applet. The second assignment was a task in the didactics course, where the participants were required to solve an Algebra problem about quadratic functions using...
different applets which the CET's site provided. The participants had two weeks to solve each assignment. They worked on the assignments in pairs or individually. The first assignment was given to the participants in the fifth and sixth weeks of the academic year, while the second assignment was given to the participants in the sixteenth and seventeenth weeks of the academic year.

The assignment that this research reports is the second assignment. In this assignment the participants were required to do the following: (1) solve an Algebra problem using the applets that accompany it (2) describe in detail their solving process, and (3) reflect on their work with the applet, emphasizing the difficulties that they met and their opinion regarding the need for applets in solving the mathematical problem they had solved.

The electronic book

The Algebra problems were part of an electronic book: Visual Math: Functions, at the CET site. This electronic book includes mathematical problems which can be solved using three types of applets.

1. An applet which is part of the problem, as figure 3 shows.

![Figure 3: an applet which is part of the mathematical problem](image)

- Construct examples of gyms for which one of the paths is longer than the other, and vice versa.
- Write expressions that describe the length of the path according to each of the two methods.
- What should be the shape of the gym if the coach’s claim is correct?

Usually this type of applets enables the user to drag the red points in the figure to change its dimensions; in our case its length, width, diagonal and perimeter.
2. An applet which is designated to be used with all assignments in a specific topic that the mathematical problem is part of. For example, the applet “The difference equation” is an applet designated to be used in solving all the assignments in the mathematical topic “solving equations”. The applet's link lies at the upper left of the web page where the mathematical problem lies. Figure 4 shows this applet.

![Figure 4: An applet for all the assignments of the topic: ‘solving equations’](image)

This applet enables the learner to write the algebraic rules of any two functions and to get their difference function algebraically, visually and dynamically.

3. An applet which designated to be used with any assignment in the whole electronic book. This type of applet is in a box called ‘Tools’. This box can be accessed by clicking a button called ‘tools’ in the upper right of any web page of the electronic book. Figure 5 shows this box.

![Figure 5: the box of tools designated to be used throughout the whole electronic box](image)

**The research tool for analyzing the data: content analysis**

I used content analysis methodology to arrive at categories of applets' functions when used to solve mathematical problems, and categories of the difficulties faced by the participants in the solving process. Content analysis determines the presence of certain words or concepts within a text in order to analyze the meanings and relationships
of such words and concepts. These meanings and relationships have usually inferences about the messages within the texts regarding the answers to the research questions.

Two experienced coders (the author is one of them) coded the participants’ solutions and reflections on their solutions, searching for categories of the participants’ perceptions regarding the role and functions of applets and of their difficulties while solving interactive mathematical problems. The agreement between the coders (Cohen’s Kappa coefficient) was 0.78 for functions of applets and 0.84 for faced problems). This type of analysis is familiar in examining perceptions of students (Wishart, 2000; Keller and Cernerud, 2002; Lizzio and Wilson, 2008)

Findings

Role and functions of applets when solving mathematical problems

The preservice teachers pointed at the following roles and functions which applets might fulfill in the mathematical problems’ context:

1. Representing the problem:
   - They help to design the story or the phenomenon in the mathematical problem.
   Preservice teacher: We started to build the playing fields that fulfill the conditions given in the problem.
   - They help students feel and see mathematical terms, objects and concepts.
   Preservice teacher: I can give this applet to middle school students to let them feel and see what areas and circumferences mean.

2. Enabling students to experiment and repeat their solution steps:
   - Applets help students repeat what they do till they see a pattern.
   Preservice teacher: I did some experiments and noticed the changes that the graphs underwent as a result of changing the functions' parameters. I continued my experiments till I arrived at mathematical relations.

3. Clarifying the story or phenomenon that the mathematical problem describes:
   - They help clarify mathematical conditions which the problem doesn't mention.
   Preservice teacher: When I read the problem I wondered what figure the hall should have, because that wasn’t mentioned in the text, but when I worked with the applet I knew that the hall should be a rectangle.

4. Influencing the nature of the mathematical problem’s solution:
   - Applets enable students to solve mathematical problems graphically or using tables of values.
   Preservice teacher: Applets let us solve mathematical equations graphically by looking at the intersection points of two functions, or finding all the x values which have the same y in the table of values.
   - Applets enable us to solve mathematical problems in different methods.
   Preservice teacher: Applets provide us with another solving method, other than the algebraic solution.

5. Applets help to evaluate one’s solution:
   - They enable the learner to check the correctness of her solution of a mathematical problem, because she can compare her algebraic solution to the one obtained from the applet.
   Preservice teacher: We decided to solve the problem in two ways: algebraically and with the applet. Doing so, we could compare between the results obtained by the two methods. This enabled us to verify the correctness of our solutions.

6. Influencing the easiness of solving mathematical problems:
   - Applets help the learner solve mathematical problems easily and in a short time.
   Preservice teacher: I didn’t expect the applet to lessen the effort of solving the problem. The solution consisted of considering three cases for every part of the problem, so I had to solve algebraically a quadratic equation for every case. I dispersed with this solving process by working with the applet. I just wrote the functions and the applet gave me the intersection points, then I continued algebraically.

7. Influencing the motivation of solving mathematical problems:
   - Applets motivate students to solve mathematical problems.
   Preservice teacher: I liked working with the applet, and this motivated me to spend a long time solving the problem.
   - Applets make solving mathematical problems enjoyable.
   Preservice teacher: I’m sure that pupils in the middle school will enjoy solving problems with applets. I learned that from my own experience.
Table 1 describes the percentage of preservice teachers who mentioned each role or function.

<table>
<thead>
<tr>
<th>Role or function of the applet</th>
<th>Percentage of preservice teachers who mentioned the function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representing the problem</td>
<td>45.45%</td>
</tr>
<tr>
<td>Enabling students to experiment and repeat their solving steps</td>
<td>10.91%</td>
</tr>
<tr>
<td>Clarifying the story or the phenomenon that the mathematical problem describes</td>
<td>7.27%</td>
</tr>
<tr>
<td>Influencing the nature of the mathematical problem's solution</td>
<td>18.18%</td>
</tr>
<tr>
<td>Helping to evaluate one’s solution</td>
<td>18.18%</td>
</tr>
<tr>
<td>Influencing the easiness of solving mathematical problems</td>
<td>21.82%</td>
</tr>
<tr>
<td>Influencing the motivation to solve mathematical problems</td>
<td>10.91%</td>
</tr>
</tbody>
</table>

We see from table 1 that the applets' function which mostly drew the participants’ attention was the ability to represent mathematical problems in ways other than algebraic.

It should be noted that the participants pointed at applets as effective tools for teaching and learning mathematics. They did so while describing the various functions of applets, like representing problems, making the solution process easier, motivating students, etc.

**Difficulties encountered by the participants while solving mathematical problems**

The preservice teachers pointed at the following difficulties faced while solving mathematical problems with applets:

1. Operating problems:
   *Preservice teacher:* It was so difficult at the beginning for me to find the intersection points of two functions. I used to look at the table of values and change the steps between consecutive x values till arriving at the intersection points. Only at the end did I realize that it’s so simple to see the coordinates of the intersection points by clicking on them.

2. Mathematical problems:
   *Preservice teacher:* I had difficulty at the beginning remembering which formula I should use in order to compute the area of the triangle in the problem.

3. Matching an applet to a mathematical problem:
   *Preservice teacher:* After reading the problem, I worked with the applet that accompanies it, but I couldn’t solve the problem with that applet, so I went to ‘Tools’ to find what applet is appropriate for solving the problem. I found that the ‘binary’ applet could solve it, but it took me much effort and time.

4. Solving a problem with applets:
   *Preservice teacher:* It was so difficult at the beginning to solve the problem using the applet that accompanies it or the applet in the left column.

5. Matching the diagram which the applet describes with the story of the problem:
   *Preservice teacher:* The applet should have had two types of boxes; one with a cover and one without a cover, but it had only a box with a cover. This made it hard for me to continue my experimentation.

6. Getting precise mathematical values from an applet:
   - Getting exact solutions
     *Preservice teacher:* I had difficulty using the applet to arrive at the exact answer of the quadratic equation because the answer was an irrational number.
     *Preservice teacher:* I spent something like ten minutes to arrive at the correct answer because every tiny dragging changed the values of the area of the playing field.

7. Constructing or drawing shapes with required dimensions:
   - Drawing a shape with specified lengths:
     *Preservice teacher:* I had difficulty drawing halls that had the given lengths because the applet didn’t enable drawing
rectangles with big areas.
8. Seeing the mathematical values which the applet shows.

*Preservice teacher*: I couldn’t see the area of the playing field because the color of the area measurement was the same as the color of the playing field.

Table 2 describes the percentage of preservice teachers who mentioned facing each difficulty.

<table>
<thead>
<tr>
<th>Difficulty faced by the preservice teachers</th>
<th>Percentage of preservice teachers who mentioned facing the difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating difficulties</td>
<td>10.91%</td>
</tr>
<tr>
<td>Mathematical difficulties</td>
<td>45.45%</td>
</tr>
<tr>
<td>Matching an applet to a mathematical problem</td>
<td>18.18%</td>
</tr>
<tr>
<td>Solving a problem with applets</td>
<td>21.82%</td>
</tr>
<tr>
<td>The matching of the diagram which the applet describes with the story of the problem</td>
<td>7.27%</td>
</tr>
<tr>
<td>Getting precise mathematical values from an applet</td>
<td>10.91%</td>
</tr>
<tr>
<td>Constructing or drawing shapes with wanted dimensions</td>
<td>14.55%</td>
</tr>
<tr>
<td>Seeing the mathematical values which the applet shows</td>
<td>14.55%</td>
</tr>
</tbody>
</table>

It should be noted that only two participants didn’t report having difficulties while using applets to solve mathematical problems.

**Methods used by the preservice teachers to solve electronic mathematical problems**

The participants used three methods to solve mathematical problems: algebraic methods, solving with applets only, blending between algebraic methods and applets.

Solving with applets:
The participants usually started solving the mathematical problem by using the applet which accompanies the problem's text. When they didn’t succeed to solve it in this way, they tried applets designated for the whole topic that their problem was part of. When they didn’t succeed too, they tried one of the applets in the Tools box.

Algebraic solution:
The participants solved a full algebraic solution in two cases: when they could not solve a problem with any available applet, or when they wanted to find out if the result obtained with applets matches the result that can be obtained algebraically.

Blended solution:
Some participants could solve with applets just part of the problem but not all of it, so they solved the rest of it algebraically. Other participants used algebra and applets together all along the solving process.

**The need for applets to solve mathematical problems**

There were four main perceptions of the need to use applets in solving mathematical problems:
1. Mathematical problems can be solved without applets, but working with them would benefit pupils in the middle school because they would visualize the mathematical problem, and thus understand it. Those who had this perception could solve the mathematical problem easily in an algebraic method but said they enjoyed working with applets. Some of them found difficulty using the applet to construct shapes with exact lengths, but nevertheless enjoyed solving the problem with applets.
2. Mathematical problems can be solved without applets, but working with them would make the solution process easier, faster and more enjoyable. Those who had this perception could solve the mathematical problem algebraically. All of them also solved the
problem in a method which blended algebra with using an applet, or which used only an applet. Some of these participants had mathematical difficulties.

3. Mathematical problems can be solved without applets.
Those who had this perception could easily solve the mathematical problem algebraically. At the same time, they couldn’t solve all parts of the problem using one of the available applets.

4. Some problems can’t be solved without applets.
Those who had this perception couldn’t solve their problems without applets. At the same time, they had difficulty understanding the mathematical problem before working with the applet.

Discussion

Niess (2006) points that to be prepared to teach mathematics, teachers need an in-depth integrated knowledge of mathematics, teaching and learning, and technology. Working with applets to solve mathematical problems, the preservice teachers built such integrated knowledge, perceiving, on the way, different roles of applets and facing different difficulties. These two factors influenced their perceptions of the need for applets to solve mathematical problems. I analyze each of these perceptions starting from the perceived role of applets.

Role and functions of applets when solving mathematical problems

To categorize the functions of applets perceived by the participants, it's possible to go after the participants who talked about applets as tools for teaching and applets as tools for learning. Using the ICT integration model of Wang and Woo (2007), regarding the three areas of ICT integration in learning, we can categorize this perception of the role of applets as accepting the applets as tools to use in the whole curriculum, not just in a specific topic. Applets were further perceived by the participants as tools for evaluating their results. Talking about applets as tools for learning, in our case solving mathematical problems, the preservice teachers emphasized the representational aspect of applets. This emphasis can be explained by the influence of different representations on the understanding of different aspects of the problem, and thus the ability to approach its solution from a better stand. This finding about the emphasis on representations is supported by the literature, for example, Santos-Trigo and Cristóbal-Escalante (2008) pointed that the use of the Cabri-Geometry software was an important tool for students to construct dynamic representations of the problems. This happened because it was used to identify and examine different mathematical relations. On the other hand, Hwang et al (2007) found that elaboration in creativity was significantly correlated with multiple representation skills. Further, the preservice teachers pointed that working with applets makes the solution process more enjoyable, so the students can be motivated to solve more mathematical problems. The enjoyment and motivation aspects could be explained by the animation aspect of applets and by the interactivity between students and applets (Crisp, 2002).

We can say that the participants perceived applets as tools that could change the solution nature of mathematical problems, and, on the other hand, as tools that enrich the problem solving context.

Difficulties faced by the participants while solving mathematical problems with applets

We can categorize the difficulties faced by the participants while solving mathematical problems using applets into: (1) Operating difficulties (2) mathematical difficulties (3) matching difficulties, and (4) compatibility difficulties. Few participants reported having operating difficulties; besides, these difficulties weren’t difficulties of inability to operate an interface element, but inability to see a ‘hidden’ element, for example, to see that by clicking on the intersection point of two graphs, one can find the coordinates of that point.

Mathematical difficulties faced by the participants were of different types: (1) understanding the story of the mathematical problem (2) understanding the conceptions of the mathematical letters that inhibit the applet (3) finding a mathematical formula which could be used to solve the problem. These difficulties emphasize the advantage of applets in the middle school, because middle school students are sure to have more mathematical difficulties than the participants, thus they can be assisted by applets to overcome their mathematical difficulties of the first and third types.
Matching difficulties were of three types: (1) finding an applet appropriate for solving a given problem (2) finding the relation between a diagram in the applet and the verbal text of the problem and (3) knowing how to solve the mathematical problem using an applet.

The first matching problem faces the student before working with an applet to solve the mathematical problem, the second matching problem faces the student at the beginning of the solving process, and the third matching problem faces the student throughout the solving process.

The matching difficulties can be explained by the short history of the participants regarding working with applets to learn mathematics, and thus they will be overcome as the learner becomes experienced in working with mathematical applets.

Compatibility difficulties were difficulties that involved the inability to get specified mathematical values or precise mathematical values using an applet, or to construct with the applet geometric shapes that have specified lengths. Generally, compatibility problems can’t be overcome by students, so they should be a challenge for software designers.

The need to use applets for solving mathematical problems

Though the preservice teachers perceived various functions and benefits of applets, as tools for solving mathematical problems, most of them thought that they can solve the problems without them. Thinking so they emphasized that, though solving mathematical problems wouldn't necessarily need applets, working with them would make the solving process easier, faster, and more enjoyable; especially for middle school students. They also pointed at the help they can get from working with applets to solve mathematical problems: "applets help us understand the problem, construct it, and have ideas how to solve it". These perceptions of the participants agree with the findings of Martínez-Santaolalla et al. (2005) who found that dynamic geometry software facilitates and enhances the learning of students.

We can categorize the perception of the participants regarding the need for applets as tools for solving mathematical problems into three categories: (1) applets aren’t necessary for solving mathematical problems (2) applets are necessary for solving mathematical problems and (3) applets aren’t necessary for solving mathematical problems, but they can enrich the solving process.

Those that considered applets unnecessary to solve mathematical problems had two history events: (1) They could solve easily the mathematical problem algebraically, but (2) they couldn’t solve some parts of the problem using only applets or in a blended method.

Those that considered applets necessary for solving mathematical problems or some mathematical problems had the following couple of history events: (1) Applets helped them throughout the entire process of solving the mathematical problem, and (2) They enjoyed working with applets to solve mathematical problems.

Those that considered applets to be unnecessary for solving mathematical problems, but considered using them as enriching the solution process, had the following couple of history events: (1) They could solve the mathematical problem working only with applets, and (2) They could easily solve the mathematical problem algebraically. Some of them also reported enjoying working with applets.

These relations between the history events and the perceptions of the preservice teachers, regarding the need for applets when solving mathematical problems, can be explained using the frame of activity theory as formulated by Leont’ev (1979).

Table 3 shows the activity of a preservice teacher, when solving a mathematical problem with applets, using Leont'ev's framework.

Those that could perform the actions and operations of the activity successfully, understanding, manipulating and utilizing the graphical signs, and enjoying this action and these operations, perceived the applets to be needed when solving mathematical problems, while those that couldn’t perform the actions and operations of the activity
successfully perceived the applets to be unnecessary for solving mathematical problems. The third party who succeeded to perform the action and operations of the activity, but succeeded easily to perform other operations that didn't follow the conditions of the activity (like working with algebraic only signs), perceived the applets as not necessarily needed but favored when solving mathematical problems.

Table 3: describing the work of the participants with applets using the activity theory frame

<table>
<thead>
<tr>
<th>Activity and Motive</th>
<th>Action and goal</th>
<th>Operations and conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Studying mathematics in an electronic book, having the motive to learn mathematics with technological tools that enable multi-representations.</td>
<td>Attempting to solve mathematical problems using applets or in a blended method: applets with algebra.</td>
<td>Working with graphic signs or both graphic and algebraic signs using applets</td>
</tr>
</tbody>
</table>

Tools are a very important component in activity theory. They mediate between the subject and the object of the activity. In our case, the subject of the activity is the student, while the object is solving mathematical problems with technological means, and the tool is the applets. We can say that the participants, perceiving the applet as a solving tool, differentiated between the following types of tools: (1) a tool that can solve problems which can’t be solved otherwise (2) a tool that can solve all mathematical problems (3) a tool that enriches the solving process but can’t solve all mathematical problems, and (4) a tool which is difficult to use to solve mathematical problems. Perceiving applets as tools from the first and the second types, the preservice teachers considered applets to be needed for solving mathematical problems. Perceiving applets as tools from the fourth type, the preservice teachers considered applets to be unneeded for solving mathematical problems, while perceiving applet as tools from the third type, the preservice teachers considered applets to be unneeded to solve mathematical problems, but working with them would help the learner during one or more stages of the solving process.

We can say that actions performed with applets, with a specific goal in mind, and their related operations, with specific conditions like the applet design, influenced the preservice teachers’ perception of applets as a cognitive tool which is needed when solving mathematical problems.

Some researchers described the importance of design issues of educational software (for example Yerushalmy, 1999; Misanchuk and Hunt, 2005). This research’s findings point at design deficiencies which can be avoided. For example, some participants claimed that some applets don’t enable the construction of geometric shapes with big values, or that the different colors of the applet's interface don’t allow seeing some measurements of the shapes in the applet. So the finding of this research can be beneficial for software designers too.

Conclusions

The participants perceived different roles and functions of applets when solving mathematical problems. They enjoyed solving problems using applets despite the fact that they faced different difficulties while doing so.

Most of the participants thought that mathematical problems could be solved without applets, but emphasized the role of applets as fostering, facilitating and clarifying mathematical problems' statement and solving. At the same time they pointed at applets as tools which learners enjoy working with, so they will be encouraged to solve mathematical problems using them. This means that the preservice teachers will probably use applets in their future teaching. They will do so when they want to foster and facilitate their students' learning or to make them enjoy the mathematical problem solving.

Factors that influenced the participants’ perception of the need to use applets for solving mathematical problems are their ability to perform successfully the activity and its related actions and operations, and the type of difficulties which hindered this performance. This says that preservice teachers should first be introduced to applets that don't have operating or compatibility problems, so they would enjoy their work with applets and not confront difficulties that lessen their confidence in applets as solving tools. This would make the preservice teachers look at applets as worth-to-work-with tools.

Designers of educational applets can benefit from this research, taking into consideration its findings, to make efficient design decisions.
References


